PROCEEDINGS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

VOL. 69 SEPTEMBER, 1943 NO. 7

TECHNICAL PAPERS

AND

DISCUSSIONS

Published monthly, except July and August, at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York, N. Y. Reprints from this publication may be made on condition that the full title of Paper, name of Author, page reference, and date of publication by the Society, are given.

Entered as Second-Class Matter, September 23, 1937, at the Post Office at Lancaster, Pa., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 5, 1918.

Subscription (if entered before January 1) $8.00 per annum. Price $1.00 per copy

Copyright, 1943, *by the* AMERICAN SOCIETY OF CIVIL ENGINEERS
Printed in the United States of America

		Discussion closes
Conformity Between Model and Prototype: A Symposium	Oct., 1942	
Discussion	Dec., 1942, Jan., Feb., Mar., Apr., May, June, Sept., 1943	Closed*
The Hydraulic Jump in Sloping Channels. *Carl E. Kindsvater*	Nov., 1942	
Discussion	Feb., May, June, Sept., 1943	Closed*
Application of Soil Mechanics in Designing Building Foundations. *A. Casagrande and R. E. Fadum*	Nov., 1942	
Discussion	Mar., June, Sept., 1943	Closed*
Flow Characteristics at Rectangular Open-Channel Junctions. *Edward H. Taylor*	Nov., 1942	
Discussion	June, Sept., 1943	Closed*
Relation of Undisturbed Sampling to Laboratory Testing. *P. C. Rutledge*	Nov., 1942	
Discussion	Dec., 1942, Mar., Apr., June, Sept., 1943	Closed*
Concrete Reservoirs of the Vertical-Beam Type. *C. Maxwell Stanley*	Dec., 1942	
Discussion	Mar., Apr., May, 1943	Closed*
Illinois Water Litigation, 1940–1941. *Langdon Pearse*	Dec., 1942	
Discussion	May, June, 1943	Closed*
Physical Properties that Affect the Behavior of Structural Members. *Wilbur M. Wilson*	Dec., 1942	
Discussion	Mar., May, June, Sept., 1943	Closed*
Organizing and Financing Sewage Treatment Projects. *Samuel A. Greeley*	Dec., 1942	
Discussion	Dec., 1942, Apr., May, Sept., 1943	Closed*
Seismic Subsurface Exploration on the St. Lawrence River Project. *E. R. Shepard and Reuben M. Haines*	Dec., 1942	
Discussion	Apr., June, 1943	Closed*
Pendleton Levee Failure. *Kenneth E. Fields and William L. Wells*	Dec., 1942	
Discussion	Dec., 1942, June, 1943	Closed*
Aeration of Spillways. *G. H. Hickox*	Dec., 1942	
Discussion	Mar., May, June, 1943	Closed*
Riveted and Pin-Connected Joints of Steel and Aluminum Alloys. *Leon S. Moisseiff, E. C. Hartmann, and R. L. Moore*	Jan., 1943	
Discussion	Apr., 1943	Oct. 1, 1943
Dewatering, Incineration, and Use of Sewage Sludge: A Symposium	Jan., 1943	
Discussion	Feb., June, Sept., 1943	Oct. 1, 1943
Primary Rôle of Meteorology in Flood Flow Estimating. *Merrill Bernard*	Jan., 1943	
Discussion	Mar., Apr., June, 1943	Oct. 1, 1943
Strains, Stresses, and Shear in Engineering Problems. *Silas H. Woodard*	Feb., 1943	
Discussion	Apr., June, Sept., 1943	Oct. 1, 1943
Determination of Kutter's *n* for Sewers Partly Filled. *C. Frank Johnson*	Feb., 1943	
Discussion	Apr., Sept., 1943	Oct. 1, 1943
Characteristics of Heavy Rainfall in New Mexico and Arizona. *Luna B. Leopold*	Feb., 1943	
Discussion	Apr., June, 1943	Oct. 1, 1943
Effect of Turbulence on Sedimentation. *William E. Dobbins*	Feb., 1943	
Discussion	Apr., May, June, 1943	Oct. 1, 1943
Development of the Chicago Type Bascule Bridge. *Donald N. Becker*	Feb., 1943	
Discussion	June, Sept., 1943	Oct. 1, 1943
Flow Around Bends in Stable Channels. *C. A. Mockmore*	Mar., 1943	
Discussion	Sept., 1943	Oct. 1, 1943
The Queens Midtown Tunnel. *Ole Singstad*	Mar., 1943	
Discussion	May, June, Sept., 1943	Oct. 1, 1943
Water Supply Engineering: Report of the Committee of the Sanitary Engineering Division for the Three Years Ending December, 31, 1942	Mar., 1943	
A Method of Computing Urban Runoff. *W. I. Hicks*	Apr., 1943	Nov. 1, 1943
Highway Engineering Education: Progress Report of the Committee of the Highway Division	Apr., 1943	
Simplified Analysis of Skewed Reinforced Concrete Frames and Arches. *Richard M. Hodges*	May, 1943	
Discussion	Sept., 1943	Dec. 1, 1943
Rôle of the Land During Flood Periods. *W. W. Horner*	May, 1943	
Discussion	June, Sept., 1943	Dec. 1, 1943
A Symmetrically Loaded Base Slab on an Elastic Foundation. *Stanley U. Benscoter*	May, 1943	
Discussion	Sept., 1943	Dec. 1, 1943
Sedimentation in Reservoirs. *Berard J. Witzig*	June, 1943	
Discussion	Sept., 1943	Jan. 1, 1944
Strength of Slender Beams. *George Winter*	June, 1943	
Discussion	Sept., 1943	Jan. 1, 1944
Sludge Drying Developments at Chicago, Ill. *Lloyd M. Johnson*	Sept., 1943	
Discussion	Sept., 1943	Feb. 1, 1944

NOTE.—The closing dates herein published are final except when names of prospective discussers are registered for special extension of time.

* Publication of closing discussion pending.

CONTENTS FOR SEPTEMBER, 1943

PAPERS

PAGE

Sludge Drying Developments at Chicago, Ill.
By Lloyd M. Johnson, Esq. 1015

DISCUSSIONS

The Queens Midtown Tunnel.
By Messrs. H. G. Cruthers, Leo Geenens, Charles E. Conover, Edward Levy, and Robert F. Legget 1025

Development of the Chicago Type Bascule Bridge.
By Messrs. Clinton D. Hanover, Jr., and Armour T. Granger 1037

Simplified Analysis of Skewed Reinforced Concrete Frames and Arches.
By Messrs. C. D. Geisler, and Arthur G. Hayden 1044

Physical Properties that Affect the Behavior of Structural Members.
By Messrs. R. L. Moore, Leon S. Moisseiff, and Fred L. Plummer 1049

Dewatering, Incineration, and Use of Sewage Sludge: A Symposium.
By Messrs. Lloyd M. Johnson, and Isador W. Mendelsohn 1055

Determination of Kutter's n for Sewers Partly Filled.
By C. E. Ramser 1063

Application of Soil Mechanics in Designing Building Foundations.
By Messrs. D. P. Krynine, O. G. Julian, A. A. Eremin, D. M. Burmister, Abraham Woolf, Lazarus White, George L. Freeman, and Trent R. Dames, Frederick J. Converse, Sterling S. Green, and Paul Baumann 1065

Strength of Slender Beams.
By Messrs. T. McLean Jasper, and C. M. Goodrich 1094

The Hydraulic Jump in Sloping Channels.
By Messrs. Frank S. Bailey, and G. H. Hickox 1097

Sedimentation in Reservoirs.
By Joe W. Johnson 1104

CONTENTS FOR SEPTEMBER, 1943 (*Continued*)

PAGE

Organizing and Financing Sewage Treatment Projects.
By A. Brinton Carson 1109

Conformity Between Model and Prototype: A Symposium.
By Martin E. Nelson 1115

Strains, Stresses, and Shear in Engineering Problems.
By Messrs. C. A. P. Turner, and Mark W. Huggins 1120

A Symmetrically Loaded Base Slab on an Elastic Foundation.
By Messrs. Robert O. Jameson, and L. J. Mensch 1125

Relation of Undisturbed Sampling to Laboratory Testing.
By D. M. Burmister 1133

Rôle of the Land During Flood Periods.
By Messrs. Raphael G. Kazmann, and W. G. Hoyt and W. B. Langbein 1138

Sludge Drying Developments at Chicago, Ill.
By C. W. Gordon 1144

Flow Around Bends in Stable Channels.
By Messrs. J. M. Robertson, and Joe W. Johnson and M. A. Selim 1147

Flow Characteristics at Rectangular Open-Channel Junctions.
By Messrs. G. H. Hickox, J. C. Stevens, and C. J. Posey 1154

TRANSLATIONS

Abridged Translations of Hydraulic Papers ..... following page 1156

(at end of vol.)

For Index to all Papers, the discussion of which is current in PROCEEDINGS, *see page 2*

The Society is not responsible for any statement made or opinion expressed in its publications

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

SLUDGE DRYING DEVELOPMENTS AT CHICAGO, ILL.

BY LLOYD M. JOHNSON,[1] ESQ.

SYNOPSIS

Since the inception of the sewage treatment program, The Sanitary District of Chicago, Ill., has conducted many investigations on the dewatering and drying of sewage sludge. This paper is concerned with these investigations, particularly the results and conclusions as applied to the Southwest works.

The Sanitary District is served by four major sewage treatment works, as indicated in Table 1. The Southwest works and the West Side works are

TABLE 1.—DATA ON THE SANITARY DISTRICT OF CHICAGO, ILL.

Plant	Design flow (mgd)	Method of sludge disposal	Date placed in operation
Calumet[a]	136	Incineration	December 3, 1935
North Side[a]	250	Pumped to Southwest works	October 28, 1928
Southwest[a]	400	Incineration	May 23, 1939
West Side[b]	472	Sand drying beds	June 2, 1930

[a] Activated sludge. [b] Imhoff tanks.

located on a common site although the treatment plants are about one mile apart. At the Southwest plant, complete treatment is provided for all the sewage from the Southwest area, and final treatment is to be provided for the partly treated effluent from the West Side works. This double function of the Southwest plant has caused it to be frequently called the West-Southwest sewage treatment plant. For the purposes of this paper it is to be identified as the "Southwest works."

The West Side works serves an area of 150 sq miles, and the population equivalent reaching the works in 1941 was 2,244,000, based on the 5-day B.O.D. analysis.

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **February 1, 1944.**

[1] Commissioner of Streets and Electricity, City of Chicago, Chicago, Ill.; formerly, Engr. of Maintenance and Operation, The Sanitary Dist. of Chicago.

The treatment plant consists of a pumping station, grit chamber and screen house, skimming tanks, and 108 Imhoff tanks with a sludge volume of 10,532,000 cu ft. Two of the batteries were designed for 134 mgd each and a settling period of 2 hr. The third battery was designed for a flow of 204 mgd and a period of 1 hr.

Digested sludge is dried on open-air beds 80 ft wide with lengths varying from 800 to 1,398 ft and having a total area of 27.3 acres. The beds are in pairs with a standard-gage railway track between each pair. Dried sludge is removed from the beds by cleaning machines that span the 80-ft width, discharging the air-dried material to 30 cu yd side dump cars. The loaded cars are hauled to dumping trestles on Sanitary District property about 3 miles west of the plant. Three heavy switching locomotives are available for hauling sludge and switching service.

The Southwest works serves an area of 82 sq miles. In 1941 the load reaching the works was equivalent to the sewage from 2,770,000 persons. The area served includes the stockyards and packing-house district.

The plant equipment includes a pump and blower house, preliminary settling tanks, aeration and final settling tanks, sludge concentration tanks, and a combined sludge-handling and steam generation plant.

The main prime movers, all driven by steam turbines, consist of four sewage pumps, three blowers, and two electric generators. Plant effluent is used as condensing water in the surface condensers of the turbines.

The sewage pumps discharge to aerated channels leading to the preliminary settling tanks. There are twelve of these tanks, each 101 ft by 103.5 ft inside, and with the depth varying from 10.66 ft to 11.66 ft. Sludge and scum removal equipment is provided. The detention period is 34 min at 400 mgd.

The aeration and final settling tanks are divided into two batteries, each having eight aeration tanks and sixteen settling tanks. The aeration units are the spiral flow type, each 434 ft long inside, and 135 ft wide, center to center of pressure walls. A depth of 15 ft over the diffuser plates is provided. Each tank is divided into four passes by three nonpressure bearing walls. The aeration period is 5 hr at 400 mgd and 20% return.

The settling tanks are the center feed, radial flow type, each 126 ft inside diameter and 11 ft deep at the side-wall. A settling rate of 1,200 gal per sq ft per 24 hr is obtained with 400 mgd and 20% return.

The plant was designed for an average flow of 400 mgd, although at present the activated sludge treatment capacity is limited by blower and sludge-drying capacity. With the present blowers a flow of 300 mgd could be treated. However, the sludge-handling capacity of about 200 tons per day, about half of which is from the North Side works, further limits the present useful capacity to about 225 mgd.

Plant extensions call for an additional battery of aeration and final settling tanks, eight additional final settling tanks for each existing battery, three additional blowers, and six more sludge-drying units. These extensions will provide activated sludge treatment for the West Side Imhoff effluent and sufficient sludge-drying capacity for the additional solids load.

The magnitude of the sludge-handling problem of the District may be visualized from the fact that at present the four major sewage treatment works

have a combined average capacity of 1,258 mgd. For 1941 the actual combined flow averaged 1,064 mgd, carrying a daily load of suspended solids averaging 729 tons, dry weight. Of this about 95% reached the Southwest works, since the North Side sludge is pumped to the Southwest works for disposal. The completion of the extensions of the Southwest works will increase the solids loads for disposal at the combined plants to more than 600 tons daily.

As early as 1914 thought was given to the dewatering and incineration of sludge by the Engineering Department of the Sanitary District. Between 1922 and 1930, full-scale tests were made at the Calumet and the Des Plaines River sewage treatment works on dewatering, drying, digestion, and other methods of disposal of activated sludge. These experiments were confined principally to various types of filter presses, dryers, centrifuges, etc. The digestion of mixed activated and fresh sludge was tried first in the operation of the West Side works. Relatively large volumes of digestion capacity and large areas of outdoor drying beds were found necessary. Difficulties in securing adequate areas, and opposition from adjoining property owners, were encountered. Consequently, a procedure was sought which could be used the year round, in which the sludge would be conditioned, dewatered to a cake, and heat dried for incineration, or placed on sale as a fertilizer. At the time, the idea of lagooning was dismissed as a temporary measure that might produce odors.

In 1932 the first tests were conducted at the West Side works on dewatering, drying, and incineration of sludge in a pilot plant handling about 20 tons of dry solids per day. Various mixtures of North Side activated sludge and West Side fresh sludge were used. In all tests the wet sludge was dewatered to approximately 80% moisture by means of a rotary vacuum filter having a net filtering area of 570 sq ft.

In the first phase of the test a rotary drier 70 ft long and 8 ft in diameter was used. Heat was supplied by an incineration furnace equipped with a stoker having a grate area of 80 sq ft.

Dried sludge from the drier was burned in the furnace along with coal, as required, to provide the necessary heat.

Vapors from the drier and the products of combustion were passed through a vapor condenser and scrubber.

The principal disadvantages of this system were:

(1) The difficulty in securing quick and responsive control; and
(2) Ineffective and inefficient deodorization of gases and vapors.

In February, 1934, the first test plant using the flash-drying system was placed in operation. Three impact mills were used for sludge, and one was provided for firing coal when supplementary fuel was required. Gases from the drying circuit were passed through the combustion zone of the furnace, and heat was recovered from furnace gases by means of a vapor heater.

The impact mills operated satisfactorily but it was found that the high degree of comminution afforded was not necessary. The mills were replaced by a cage mill with an evaporating capacity of 5,000 lb per hr of water. A sub-

stantial saving in power was realized by this change. Changes were also made to permit use of a vapor cycle instead of an air cycle in the drying circuit.

The tests in this plant were completed in August, 1935. The Calumet Sewage Treatment Works, having a capacity of 40 tons of dry solids per day, was the first plant built after these tests. Experience gained from the operation of this plant aided in the design of the Southwest works. However, the Southwest sludge handling plant is more complicated than the Calumet plant, as the function of steam generation is combined with that of sludge drying.

In planning the sludge-handling system at the Southwest works, provision was made for handling the North Side sludge, in amount about 100 tons of dry solids per day. The North Side sludge and the Southwest activated sludge are mixed, screened through mechanically cleaned bar screens with ½-in. clear openings, and passed to six concentration tanks, each 70 ft by 46 ft 9 in. inside measurements. A centrifugal pump is provided at each tank to pump the concentrated sludge to the sludge-handling building. The supernatant liquor is discharged to the incoming sewer by gravity.

The concentrated sludge is pumped into the conditioning apparatus on the top floor of the sludge disposal building. The sludge is coagulated by adding a concentrated solution of ferric chloride in a dosing flume. The flow of ferric chloride is proportioned to the flow of sludge by the operator as required by changing sludge conditions. From 6% to 9% of ferric chloride is used, based on the dry weight of the solids. The best guide is the pH-value of the conditioned sludge, which is recorded in a continuous graph. From the mixing flume the sludge is discharged directly to the vacuum filters.

There are thirty-two vacuum filters, each with a drum 11.5 ft in diameter and 16 ft long, with drain valves at each end (see Fig. 1). The filters are arranged in groups of four, each group discharging cake to a common conveyer belt leading to one drying unit. A constant effort is being made to reduce the costs of dewatering. Various substitutes are under investigation to replace the monel metal mesh for supporting the filter cloth on the outside of the drum. The use of plastics for interior piping in place of copper pipe and bronze fittings is being tested. Stainless steel has been found best adapted for the scrapers, although plastics are to be tried as a substitute. The most satisfactory filtering medium is a long-nap, virgin-wool cloth, weighing 13 oz per sq yd. The substitution of a coated iron wire for bronze or copper as winding wire over the cloth is under trial. The cake is peeled off the filters at 80% to 85% moisture, depending on the type of sludge. The sludge will filter more readily to a lower moisture as its volatile content decreases.

As the cake moves along on the belt conveyer, it is weighed automatically. It is then mixed with previously dried sludge to produce a mixture containing between 40% and 50% moisture. The mixing is done in U-shaped troughs lined with stainless steel in which paddles mounted on horizontal shafts mix the wet and dry sludge and move it along the trough. In these units severe abrasion occurs, particularly at the tips of the paddles. The use of a chromium-tungsten-cobalt steel alloy reduces the wear. The mixing has been improved by reversing some of the paddles on each shaft. The mixed sludge is passed to the drying units.

Each of the eight sludge-drying units has a normal capacity to evaporate 20,000 lb per hr of water, based on 1,150° F vapor inlet temperature and 300° F outlet temperature. The drying units operate at a pressure slightly below atmospheric to prevent escape of odors from the process. As a further assurance that no odors will be released to the atmosphere, all waste vapor is passed

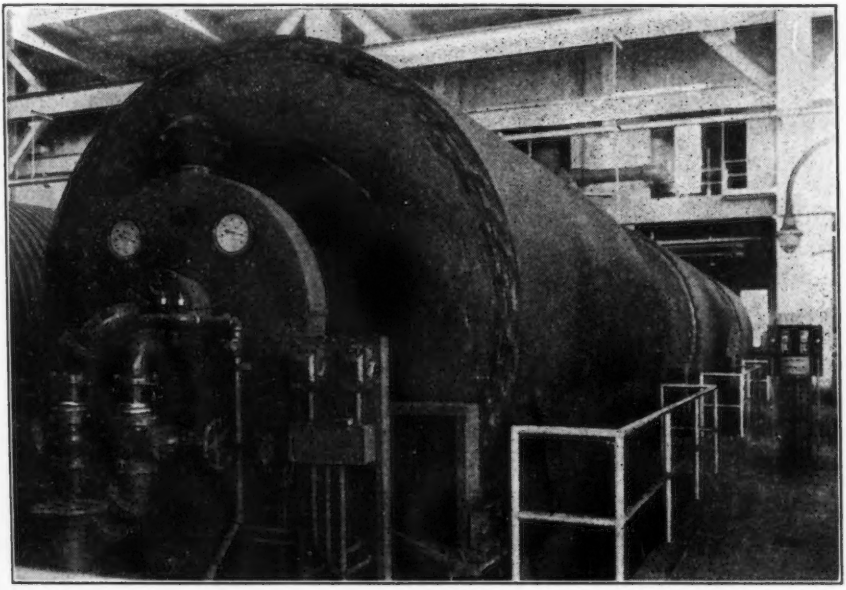

FIG. 1

through the combustion zone of the furnace. The mixed sludge from each group of four filters is divided between the two halves of the drying tower for each drying unit, the division being controlled by the vapor temperature in the cyclone separators. Each tower is about 12 sq ft in total cross-sectional area. A 3-in. thickness of insulating block is used, lined with stainless steel containing 18% chrome and 8% nickel. It is here that the sludge is mixed with the superheated vapor from the vapor heater.

At the bottom of the towers, the sludge and vapor mixture enters a squirrel-cage mill, which forms the flash drier. Each drier unit consists of two squirrel-cage mills mounted on a common shaft. The mill breaks up the sludge and brings the particles into intimate contact with the superheated vapor, resulting in very rapid drying. By the use of flash drying, cake at 80% moisture can be dried to 5% moisture in 5 min, as compared with 90 min in a modern rotary drier. The driers are subject to abrasion, but as yet this has not been serious. The wear is confined mainly to the liner plates and the cage bars. The renewal of cage bars has been made less frequent by the use of removable sleeves of alloy steel.

Each half of the sludge drier discharges to a cyclone separator from which the dried sludge is removed through rotary air locks and passed to a surge bin.

The vapor passes from the separator to the circulating fan and thence to the vapor heater and through the circuit again.

The abrasion on the liner plates of the separators is serious. The original plates were made of ¼-in. steel and were very short lived. The separators were then relined by the gunite method, using lumnite cement and crushed trap rock (ganister) reinforced by No. 10 galvanized steel mesh with 3-in. square openings. This lining has proved very satisfactory.

During the first few months of operation the circulating fans required attention because of a deposit of grease and solids on the blades, which unbalanced the units. Deposits also formed in the ducts. This difficulty was reduced by diverting all preliminary tank solids to the West Side works, and handling only the activated solids in the drying plant.

The vapor heaters are subjected to high temperatures and require close supervision in operation. Room air is used as a tempering medium to hold the temperature of the inlet gas between 1,475° and 1,500° F. Particles of finely divided sludge may deposit on the plates during passage through the vapor side of the heater and may ignite if temperatures in the gas side become too high. The units are protected by thermostatically controlled steam valves which open into the hot gas duct if the temperature becomes excessive.

The surge bins receive the dried sludge from the cyclone separators and serve as distributors for supplying dried sludge to the sludge mixers and to the sludge burners or to the fertilizer conveyer. These units are insulated to prevent condensation on the walls.

Water-cooled butterfly dampers have been installed in the hot gas ducts between the furnaces and vapor heaters. The condensate from the sewage pump and blower turbines is used as a cooling medium, after which it is discharged to the de-aerating heater.

The heat required by the drying system is drawn from the four furnaces, each serving two drying units. They are of the water wall type, fired vertically through four burners for dried sludge and six burners for pulverized coal. They were designed for supplying heat under normal conditions to evaporate 40,000 lb per hr of water from sludge and also to generate 70,000 lb per hr of steam at 425 lb per sq in. gage and 750° F, total temperature, with feed water at 275° F. The boilers are the bent tube type with two drums. A superheater is provided.

The functions of steam generation and sludge drying were combined, not primarily with the recovery of heat from burning sludge as the end in view, but because most of the operating personnel and some of the physical structures of a steam generating plant are already provided in the sludge-drying plant. The only additional cost items for steam generation are fuel and fixed charges on steam generating equipment. Clarified plant effluent is used for condensing purposes. The ample supply has sufficient head to provide gravity flow and to eliminate the need for circulating pumps.

The waste furnace gases pass through an air heater, wherein the combustion air is preheated. After passing through this unit the gas mixes with the furnace gas withdrawn by the drying system. It passes through electrical dust precipitators and then to the stack. There are two dust precipitators, each rated at 180,000 cu ft per min of gas.

With fuel requirements dependent on two variables—steam load and drying load—the control of the furnaces is a delicate problem. Automatic control equipment is provided which ties in the control of the furnaces and drying systems.

In the tuning-up period, better control of steam temperature was secured by moving the superheater by-pass damper from near the top of the front baffle to the bottom of the by-pass passage. Furthermore, to secure higher superheated steam temperatures during high drying and low boiler loads, the sidewall tubes were covered with a plastic refractory, which permits of higher furnace temperatures while reducing the heat absorbed by the water walls, thus increasing the heat passing through the main body of boiler tubes and the superheater.

Operating experience indicated the desirability of changing the control for the flow of combustion air. Originally the control acted through totalizing impulses from the flow of steam and waste vapor, which changed the position of the damper of the forced draft fan. This was altered to control the position of the damper by the rate of coal feed, balanced by the differential pressure across an orifice in the forced draft fan discharge duct. The new arrangement insures a definite quantity of air regardless of changes in the position of the secondary and tertiary air dampers.

A considerable amount of experimenting was done on the arrangements of secondary air ports. It was found necessary to reduce the number of ports and the total area of the openings to produce sufficient turbulence to insure complete combustion of the fuel.

Coal is delivered to the plant by rail to a track house south of the sludge disposal building. At this point it may be elevated to a conveyer leading to the bunkers or placed in storage in an adjacent area. A coal crusher is provided which may be used before the coal is elevated, if necessary. A belt conveyer with a traveling tripper distributes coal to the bunkers. This equipment has a capacity of 100 tons per hr. Each furnace is provided with a bunker holding 300 tons.

Coal for each furnace is drawn from the bunkers through two lines, each being equipped with a scale and feeder. Each furnace is served by two pulverizers of the bowl mill type with direct-connected exhausters.

When the firing equipment was designed and purchased the immediate production of fertilizer was not intended. Hence, with the entire sludge output being sold as fertilizer the pulverizing capacity provided is inadequate, since the heat value of each ton of fertilizer produced is equivalent to about 0.57 ton of coal. Also, the elimination of the preliminary solids presents a higher filter-cake moisture than would be the case with mixed solids, thus requiring more heat for drying.

Bottom ash collects in hoppers in the furnace bottom, below the superheater pass and below the boiler back pass, and also in the bases of the stacks. It is removed by hydraulic ash sluicing equipment and sluiced to a common sump, from which it is pumped to a dewatering hopper in the track house or to an ash lagoon. Ash pumped to the track house is loaded into railway cars and hauled to the sludge dumping grounds.

Fly ash is collected in hoppers below the air preheaters and dust precipitators. It is removed by a vacuum system discharging to storage hoppers in the track house. The material is moistened sufficiently to prevent blowing as it is drawn from the hoppers and then loaded into railway cars for hauling away from the plant.

Conclusion

Sludge disposal methods in use at the various works of The Sanitary District of Chicago were developed over a period of many years of operating experience and experimental investigation. Through the development of the flash-drying process, as installed at the Calumet and Southwest works, a method of handling sewage solids without the production of offensive wastes and odors was obtained. Although it is considered by those charged with the responsibility of design and operation that these installations have functioned, and will continue to function, in a very satisfactory manner, further improvements will no doubt be evolved that will decrease the cost of sludge disposal. There have been numerous problems attendant upon the operation of this new process, but in spite of these problems the method has proved its value as evidenced by the Sanitary District in enlarging the Southwest works along similar lines.

TABLE 2.—Operations Results, 1941, Major Sewage Treatment Works

Treatment works	Flow (mgd)	Raw Sewage (ppm)		Effluent (ppm)		Solids removed; dry basis (tons)
		5-day B.O.D.	Suspended solids	5-day B.O.D.	Suspended solids	
Calumet	66.4	95	130	9	12	11,836
North Side	206.8	116	146	5.9	9	40,529
Southwest[a]	338	164	205	16[b]	17[b]	67,405
West Side	453.0	131[c]	198[c]	73	90	38,249

[a] Preliminary treatment only, 184 mgd. Activated sludge treatment, 154 mgd. [b] Activated sludge effluent. Weighted average of activated sludge effluent and preliminary tank effluent by-passed 72 ppm, both B.O.D. and solids. [c] Analysis includes preliminary tank sludge from the Southwest works.

TABLE 3.—Sludge-Handling Plants, Calumet and Southwest Sewage Treatment Works, 1941

Treatment works	Percentage of Dry Solids			Fuel value of dried sludge (Btu per lb)	Water evaporated (lb)
	Filter cake	Mixed sludge to driers	Dried sludge		
Calumet	17.3	73.4	93.9	5,920	95,380,000
Southwest	16.6	56.2	95.0	6,888	581,474,000

Although this paper deals chiefly with the Southwest sludge handling plant, the magnitude of the sludge problems at the West Side works must not be overlooked. In 1941 the Imhoff tanks removed a total of 76,897 tons of dry solids, an average of 211 tons per day. The volume of wet digested sludge removed in 1941 was 644,055 cu yd. Of this amount 78,099 cu yd were dis-

charged to temporary lagoons on the plant grounds, the remainder going to the drying beds.

An additional load was thrown on the plant in 1941 by the diversion of preliminary sludge from the Southwest works. This amounted to 132 tons of dry solids per day.

The capacity of the sludge-drying beds is inadequate even though worked to the limit. At times the air-dried sludge removed is rather high in moisture. The average for 1941 was 65.1%, compared to the previous maximum annual average of 54.0%. However, the sludge strippers handle this wet material easily.

Operating results for the year 1941 at the four major works of the Sanitary District are presented in Table 2. In Table 3 data on operation of the sludge-handling plants are presented for the same year.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE QUEENS MIDTOWN TUNNEL

Discussion

BY MESSRS. H. G. CRUTHERS, LEO GEENENS, CHARLES E. CONOVER, EDWARD LEVY, AND ROBERT F. LEGGET

H. G. CRUTHERS,[12] ESQ.[12a]—The cost of 0.1¢ per vehicle, to provide power for ventilation for the vehicles carried by the Queens Midtown Tunnel, as shown in Mr. Singstad's paper on that project, is extremely low compared to that for similar tunnels and reflects not only a high over-all efficiency for the ventilating equipment but a low cost of power.

For the first year of operation, the actual cost of power for all purposes was 0.9548¢ per kw-hr. This cost included all charges for energy, demand, and fuel and was made possible through purchase from the New York City pool.

The power used for ventilation during 1941, when 4,400,000 vehicles more or less were carried by the Queens Midtown Tunnel, was approximately 0.111 kw-hr for each vehicle. From data available, the power required to provide ventilation for the Holland Tunnel during 1936, when 11,499,000 vehicles were carried by that tunnel, was approximately 0.529 kw-hr per vehicle and for the Lincoln Tunnel during 1938, when 1,790,000 vehicles were carried by that tunnel, approximately 0.289 kw-hr per vehicle.

It has been estimated that, when the Queens Midtown Tunnel carries 11,499,000 vehicles per year, the power required for ventilation will be about 0.10 kw-hr per vehicle.

The number of possible air changes provided for each of these three tunnels is identical (42 per hour) so that the difference in the amount of power required per vehicle carried depends on the method of providing the air changes.

The Holland Tunnel (two tubes) is divided into fourteen ventilating sections, the average length of each being approximately 1,200 ft. The Lincoln Tunnel, a single tube, is divided into five ventilating sections, the average length of each being 1,640 ft. The Queens Midtown Tunnel (two tubes) is divided into eight ventilating sections, the average length of each being 1,590 ft.

NOTE.—This paper by Ole Singstad, M. Am. Soc. C. E., was published in March, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1943, by Messrs. Thomas W. Fluhr and Howard L. King; and June, 1943, by Messrs. Francis V. Wagner, William J. Wilgus, and S. A. Thoresen.

[12] Mechanical Engr., New York City Tunnel Authority, New York, N. Y.

[12a] Received by the Secretary June 3, 1943.

The areas of the air ducts of the Lincoln Tunnel and the Queens Midtown Tunnel are approximately, for the fresh air ducts, 25%, and for the exhaust air ducts, 20%, greater than those of the Holland Tunnel.

Since the power required to provide ventilation by the transverse method of a continuous rate of supply and exhaust varies inversely as the square of the areas and directly as the fourth power of the lengths of the ducts, it would appear that the power required to provide ventilation for each vehicle carried by either the Lincoln Tunnel or the Queens Midtown Tunnel should be much greater than the power required at the Holland Tunnel. The data show that this is not so—0.529 kw-hr per vehicle at the Holland Tunnel, 0.289 kw-hr per vehicle at the Lincoln Tunnel, and 0.111 kw-hr per vehicle at the Queens Midtown Tunnel. The reason for the lower values for the Lincoln Tunnel, as well as for the Queens Midtown Tunnel, is attributed to the use of higher motor efficiencies at the low-speed operation of the ventilating equipment.

Records from the Holland Tunnel for 1936 operation show that the ventilating equipment was operated for 90% of the time at fan speeds which supplied 29.5% of the normal maximum requirements. Maximum requirements are satisfied by the use of two fans on a duct operating at full speed, and 29.5% of this was obtained at the Holland Tunnel by one fan on a duct operating at approximately 48% of full speed.

The records of operation also indicated that for a great part of the time the ventilation requirements were such that much less than 29.5% of maximum requirements would have been ample to provide good atmospheric conditions in the driveways so that, if the ventilating equipment of the Holland Tunnel could have been operated to provide something less than 29.5%, greater economies could have been effected.

With these records of operation as a guide, the Queens Midtown Tunnel was provided with equipment which would allow for operation such that one fan could be operated on a duct at one-third full speed, supplying approximately 20% of maximum ventilation requirements.

At this tunnel, as described by Mr. Singstad, dual-motor drives were installed to operate the fans—one large single-speed motor to drive the fan at full speed and one small two-speed motor to drive the fan at two-thirds' full speed and at one-third full speed. These motors were all totally enclosed fan-cooled squirrel-cage motors with average efficiencies of 92.5% for full speed, 90.7% for two-thirds of full speed, and 85.2% for one-third of full speed, the latter to operate for more than 90% of the time.

At the Holland Tunnel, one slip-ring, wound-rotor motor with external resistance for varying the fan speed was installed to drive each fan. The average efficiencies of the motors were as follows: For full-speed operation, 91%; for 87% of full speed, 78%; for 72% of full speed, 64%; and, for 48% of full speed, 38%.

The difference between motor efficiencies at the low-speed operation of 85.2% for the Queens Midtown Tunnel and 38% for the Holland Tunnel, together with the fact that for a great part of the time ventilation requirements are such that 20% of maximum requirements instead of 29.5% are sufficient to maintain good atmospheric conditions in the driveway, accounts in a large

part for the much lower power requirements per vehicle carried—0.111 kw-hr for the Queens Midtown Tunnel as compared to 0.529 kw-hr for the Holland Tunnel.

Dual-drive, squirrel-cage motors also were installed to drive the fans at the Lincoln Tunnel. These motors had operating efficiencies comparable to those installed at the Queens Midtown Tunnel, but the average lengths of ventilating ducts at the Lincoln Tunnel were greater than those of the Queens Tunnel, and, since the Lincoln Tunnel is a single tunnel for two-way operation, the traffic conditions impose a greater load factor; and it is presumed that these two conditions alone account for the difference between the power requirements of 0.289 kw-hr per vehicle at the Lincoln Tunnel as compared to 0.111 kw-hr per vehicle at the Queens Midtown Tunnel.

LEO GEENENS,[13] ESQ.[13a]—The author reports a substantial reduction in the cost of tunnel operation. Approximately three quarters of the operation cost represents wages, mostly of policing, toll-collecting, and maintenance personnel. In general, the greater part of the savings in operating the Queens Midtown Tunnel as compared with the Holland Tunnel is the result of reducing payroll costs. In a large measure this was accomplished through skillful designing based on past operating experience.

During the design period, the selection and location of apparatus, the selection of materials, and the arrangement of rooms in the buildings were studied in relation to the organization chart for operation adopted at the start of the project. In this chart the personnel was reduced drastically. Man-hours could not be decreased by the elimination of any major operating function but rather by their curtailment on a very large number of relatively small operations. The following are a few examples of the saving obtained in this way in the electrical and mechanical fields:

1. Where possible, operations were made automatic; such was the use of automatic dial telephones to eliminate telephone switchboard operators. In other cases, single operations were made simpler and easier in order to permit employees to take on additional duties—for instance, in the case of the operators attending the main ventilation and traffic control switchboard. This function was combined with the administrative work of the police force and resulted not only in saving man-hours but also in producing a smoother working force with more centralized authority.

2. A reduction in the electrical and mechanical personnel was effected by judicious selection of the apparatus in order that it might function safely without attention for long periods of time. As many as eight operators per shift or a total of thirty-two men (counting relief men) were eliminated in this way. Also the number of maintenance men needed for fans, motors, and pumps was reduced.

3. Particular attention was given to the ever-present problem of corrosion, as painting is a large item of maintenance. Where economically possible, non-

[13] Electrical Engr., New York City Tunnel Authority, New York, N. Y.

[13a] Received by the Secretary June 28, 1943.

ferrous metals or materials were used. For instance, as mentioned in the paper, asbestos cement boards were used for baffles and louvres in the ventilation system; all bolts, nuts, screws, washers, and small parts in electrical devices are of silicon bronze; electrical conduits, below ground, are silicon bronze or asbestos cement; electrical relays are sealed in glass containers; motors are totally enclosed; etc.

Many similar reductions were made. For instance, maintenance labor was reduced by a radical simplification of the carbon-monoxide analyzers and recorders. Savings in man-hours have resulted from improved toll-registering machines which have reduced the auditing time. For washing the tile walls in the tunnel, a new machine which does the work faster and with less labor was designed. In order to decrease payroll costs, every operating function was analyzed in the greatest detail.

CHARLES E. CONOVER,[14] M. AM. SOC. C. E.[14a]—Under the heading, "Historical," Mr. Singstad states: "In July of the same year [1929], the Board of Transportation was assigned the work of preparing plans and specifications [for the Queens Midtown Tunnel] and starting construction with an initial appropriation of $2,000,000."

As all who have studied vehicular tunnels know, their success or failure depends largely on the plaza layouts together with the approaches on the assumption, of course, that the development leading up to the plazas warrants the expense of tunnel construction. By approaches is meant not only the streets in the immediate vicinity, but also the traffic arteries that may extend for miles beyond. To illustrate: In the case of the Holland Tunnel the New York City approaches are limited very largely to the lower and midtown areas of Manhattan via adjacent streets and to Brooklyn via Manhattan Bridge, whereas the New Jersey approaches may be considered to extend to, and connect with, every municipality in central New Jersey and even Philadelphia, Pa.

The location of any expensive vehicular traffic facility, whether bridge or tunnel, must be the subject of extensive preliminary investigation, and, as Mr. Singstad has indicated, the Regional Plan Association had made very extensive and comprehensive traffic studies covering midtown Manhattan especially. This research clearly showed the great need, as Mr. Singstad so aptly states, for an east and west traffic axis from Long Island to New Jersey.

The investigations and the plans prepared by the Board of Transportation led to the conclusion that, to render the maximum benefit to Manhattan and Queens and to serve the entire metropolitan area most advantageously, such a traffic axis should have ample and convenient plazas on both the east and west sides of Manhattan and should extend by fairly direct connections far into New Jersey and Long Island. It should provide for express highway service free of the shackles that "stop-and-go" lights impose on fast and continuous service. In practical terms the East River tunnel should so connect with the Hudson River, or Lincoln Tunnel (then being studied, and later constructed

[14] Cons. Engr., Pearl River, N. Y. (formerly Div. Engr. of Planning and Design, Board of Transportation, New York, N. Y.).

[14a] Received by the Secretary July 13, 1943.

and operated, by the Port of New York Authority), that traffic between New Jersey and Long Island could operate across Manhattan without emerging into the street system or in any way interfering with traffic.

Through the splendid cooperation of the engineers of the Board of Transportation with the engineers of the Port Authority, a joint west side plaza was developed, and, through the cooperation of the engineers of the Board of Transportation with the engineers of the Borough of Manhattan, a joint east side plaza was developed.

Although the east side plaza, or the Manhattan plaza, of the New York City Tunnel Authority differed radically from the east side plaza as developed by the Board of Transportation, it retained the possibility of a direct connection with the west side plaza (the Manhattan plaza of the Lincoln Tunnel), and thereby provided the possibility of a direct New Jersey-Long Island highway, if and when the Manhattan tunnels were constructed.

It is to be hoped that the New York City authorities will plan these connecting tunnels as a postwar project. With that connection there should be considerable lessening of crosstown traffic in the midtown Manhattan area, and a considerable increase not only in the traffic of the Queens Midtown Tunnel but also in that of the Lincoln Tunnel. This trend is indicated clearly by the traffic studies of the Regional Plan Association. Further extension into and through New Jersey by express highway or highways also would increase the traffic and relieve local congestion.

In addition, the plans of the Board of Transportation provided an express connection extending several miles into the Borough of Queens, somewhat similar to that mentioned in the paper. That connection should be constructed as a postwar project, as should a viaduct from the Queens plaza crossing Newtown Creek and connecting with the Brooklyn crosstown highway which was under consideration in 1929 and 1930. These connections not only would increase the traffic through the East River tunnels, but would relieve, most assuredly, traffic at and near the Queens plaza.

A detail that should be studied, if and when the Manhattan tunnels are constructed, is the method of connecting those tunnels to the north and south auxiliary streets provided as a part of the Manhattan plaza of the Queens Midtown Tunnel, so that the traffic may move in one direction if reasonably possible. In other words, the auxiliary street east of Third Avenue should be an exit street, not only for the river tunnels, but also for the Manhattan tunnels. Likewise, the auxiliary street east of Second Avenue should be an entrance street for both facilities. The purpose of such an arrangement would be to simplify the flow of traffic at the intersections of the auxiliary streets with cross streets.

In the case of the Queens Midtown Tunnel, the lighting that was developed for the portals where special attention was given to the difficulty of passing between brilliantly sun-lighted plazas on clear days and the more dimly artifically-lighted tunnel portals undoubtedly has resulted in better driving conditions at the portals, than at the portals of the Holland Tunnel.

Under the heading, "Introduction," Mr. Singstad refers to "remote control of ventilation." It is suggested that the designers might have gone a step

further and provided not only remote control, but automatic control, of ventilation.

A detail, not mentioned in the paper but which may have been considered, is adequate widening and suitable arrangement at curves to provide for the passage of fire-fighting apparatus. In the plans prepared by the Board of Transportation this matter was given careful attention.

EDWARD LEVY,[15] M. D.[15a]—The same high standards of achievement were set by Mr. Singstad for the control of safety and health of the men working under compressed air on the Queens Midtown Tunnel, as for the other features of this outstanding project. The experiences on the Queens Midtown Tunnel in this field have provided considerable important data which may help to establish improvements in methods affecting health, safety, and economy on future work.

As stated by the author, the medical record for work in compressed air on the Queens Midtown Tunnel shows great improvement over that of the East River tunnels of the Pennsylvania Railroad. These railroad tunnels were built at a time (1904–1908) when practical regulations for the prevention of compressed air illness were still in an experimental stage.[16] As shown in Fig. 2, the Queens Midtown Tunnel is located near the Pennsylvania Railroad tunnels, and, consequently, conditions encountered in construction were similar in many respects on the two projects. The comparison, denoting progress in the control of compressed air illness during the period from 1904 to 1939, is therefore a sound one, and the improvement shown is undoubtedly due to better regulations and better enforcement.

There were no legal regulations during the construction of the Pennsylvania Railroad tunnels. For gage pressures up to 32 lb per sq in., the working day was 8 hours long, with a half-hour rest period. For gage pressures of from 32 to 42 lb per sq in., there were two 3-hr work periods per day with a 3-hr rest period between them. An attempt was made to enforce a maximum decompression rate of 2 lb per min.

The first statutory regulations for work in compressed air to be adopted by New York State became effective on January 1, 1910. These have since been modified from time to time by reducing the hours of work. Projects constructed since 1925, including the 53d Street, Fulton, Rutgers-Jay, and Harlem River tunnels and part of the Holland Tunnel work, have been under the regulations shown in Table 3. The rest periods are ones actually adopted and are equal to or greater than the prescribed minimums.

The maximum rates of decompression required by these regulations are: For 0 to 15 lb per sq in., 3 lb per min; for 15 to 20 lb per sq in., 2 lb per min; for 20 to 30 lb per sq in., 3 lb per 2 min; and for 30 lb per sq in. or over, 1 lb per min. A stage method of decompression is used in which the pressure in "locking out" is dropped at a rapid rate (specified at 5 lb per min) to half the gage pressure, and the remaining decompression is at a uniform rate such

[15] Medical Consultant, New York City Tunnel Authority, New York, N. Y.

[15a] Received by the Secretary July 23, 1943.

[16] "Caisson Disease and Its Prevention," by Henry Japp, *Transactions*, Am. Soc. C. E., Vol. 65 (1909), p. 1.

that the total time of decompression, including that for the initial rapid drop in pressure, shall equal the time specified for the original maximum pressure.

The Queens Midtown and Lincoln tunnels both were built under these regulations with the modification that the schedule of 3 hours of work, 3 hours of rest, and 3 hours of work was adopted at 0 to 18 lb per sq in. as well as in

TABLE 3.—REGULATIONS IN EFFECT IN THE STATE OF NEW YORK, GOVERNING THE MAXIMUM DAILY HOURS OF WORK PERMISSIBLE UNDER COMPRESSED AIR

Time sequence (hours)	GAGE PRESSURE, IN POUNDS PER SQUARE INCH						
	0–18	18–26	26–33	33–38	38–43	43–48	48–50
Work	4.0	3.0	2.0	1.5	1.0	0.75	0.5
Rest	0.5	3.0	4.0	4.5	6.0	5.50	7.0
Work	4.0	3.0	2.0	1.5	1.0	0.75	0.5

the higher range from 18 to 26 lb per sq in. Also, on the Queens Midtown Tunnel, the decompression times in the higher pressures were increased in length from 2 to 8 min above those required by specified maximum rates.

The Clark, Montague, 60th Street, and 14th Street tunnels were built under statutory regulations that permitted 8 hours of work in 24 hours in gage pressures of from 0 to 22 lb per sq in., two 3-hr work periods in from 22 to 30 lb per sq in., two 2-hr work periods in 30 to 35 lb per sq in., two 1.5-hr work periods in 35 to 40 lb per sq in., two 1-hr work periods in 40 to 45 lb per sq in., and two 0.75-hr work periods in 45 to 50 lb per sq in. Decompression was specified to be at the maximum rate of 3 lb per 2 min up to 36 lb per sq in., and 1 lb per min in pressures of 36 lb per sq in. and greater. Part of the Holland Tunnel work was performed under this schedule of hours of work with the exception that the first and second ranges of pressures were 0 to 21 and 21 to 30 lb per sq in., respectively.

It is noteworthy that there have been no deaths due to compressed air illness since the construction of the 60th Street tunnel, built from 1917 to 1921. In fact, the record shows quite conclusively that, although legislation is periodically demanded to further shorten the lengths of the working shifts, a practical limit already has been reached if not surpassed in this regard. Naturally it is to be expected that the shorter the time of exposure in compressed air the less the incidence of compressed air illness will be, but this can be carried to a point at which the cost of compressed air tunneling would be prohibitive. Although the practice, on the Queens Midtown Tunnel and other recently constructed tunnels, of limiting the working time to a maximum total of 6 hours in 24 hours is sound both from medical and operational standpoints, it is evident from the record that further reduction in the length of the working shift is not warranted.

Work under compressed air at the Queens Midtown Tunnel was supervised most carefully, and, in some cases, hourly limitations of work and decompression requirements were even more rigorous than required by the code. As stated

by the author, working conditions and safety precautions were strictly in accordance with the best standard practice. There were almost three quarters of a million decompressions, most of them from pressures of from 26 to 37.5 lb per sq in. The reporting of cases of compressed air illness was encouraged and the records were kept carefully. It was the intention to approach the ideal as closely as possible. In this the Tunnel Authority had the full cooperation of the contractor and his staff, and also that of the members of the Compressed Air, Tunnel, Caisson, Subway, Cofferdam and Sewer Construction Workers Union.

Having in mind the idea of using oxygen while decompressing, so as to hurry the desaturation of the body fluids and tissues of excess nitrogen, the help of the Division of Industrial Hygiene of the National Institute of Health, U. S. Public Health Service, and U. S. Bureau of Mines was enlisted in a field study along these lines conducted from July, 1938, to May, 1939.

Theoretically, at least, this use of oxygen should have been successful. In conjunction with the investigation,[17] blood and urine tests were made on men who had suffered from attacks of compressed air illness, but no causative reason was found to show why these men should have been attacked while others, in whom there was no physiological difference, were not. Briefly, the tests with oxygen breathing were successful when conducted with smaller groups under the direct supervision of the representatives of the government agencies. When continued on a more extensive scale under the contractor's supervision, they were somewhat disappointing. The investigation indicated that the use of oxygen in decompressing would be practicable for the smaller groups of men, especially when they had been subjected to the higher pressures.

As a by-product of the oxygen-breathing tests conducted on the Queens Midtown Tunnel work, the use of helium-oxygen mixtures for the alleviation of ear block was developed successfully.[18] As ear block is the cause of many compensation cases on compressed air work, as well as sometimes the cause of delay when a gang is locking into the tunnel, this use of a helium-oxygen mixture (for which a practical apparatus was perfected on the Queens Midtown Tunnel) is an important development.

It is interesting to note that on the Queens Midtown Tunnel work the onset of compressed air illness occurred in 38% of the cases during the first half hour after decompressing, 54% occurred during the first hour, 66% in the first hour and a half, and 72% during the first two hours after decompression. Corresponding percentages based on the record of a great number of cases on previous projects are, respectively, 44%, 64%, 73%, and 82%.

A comparison of the percentages of types of symptoms on the Queens Midtown Tunnel (A), the Pennsylvania Railroad East River tunnels (B), and 680 cases of compressed air illness in New York Public Service Commission Tunnel projects (C) is afforded by Table 4. These percentages are fairly

[17] "Administration of Pure Oxygen to Compressed Air Workers During Decompression," by R. R. Jones, J. W. Crosson, F. E. Griffith, R. R. Sayers, H. H. Schrenk, and Edward Levy, *Journal* of Industrial Hygiene and Toxicology, Vol. 22, No. 10, December, 1940, pp. 427–444.

[18] "Helium-Oxygen Mixtures for Alleviation of Tubal and Sinus Block in Compressed Air Workers," by J. W. Crosson, Roy R. Jones, and R. R. Sayers, *Public Health Reports*, Vol. 55, No. 33, August 16, 1940, pp. 1487–1496.

consistent. As the joints, fibrous tissues, and fats of the body (which have poorer circulation than other parts of the body) desaturate more slowly during decompression, this accounts for the high percentage of symptoms which are classified as localized pain in the body.

TABLE 4.—INCIDENCE OF SYMPTOMS (PERCENTAGE OF TOTAL CASES)

Type of Symptom	A	B	C
Central nervous system	2.7	2.2	1.6
Localized pain in body	86.2	90.3	91.7
Vertigo	9 9	5.4	6.5
Chokes	1.0	1.6	0.1
Partial or complete unconsciousness, with collapse	0 2	0.5	0.1
Totals	100.0	100.0	100.0

A study of the blood of compressed air workers on the Queens Midtown Tunnel gave no clew to the susceptibility of some workers over others. However, there was a definite feeling that alcohol played a part in the occurrence of compressed air illness. Grover C. Emery, M.D., who was in direct medical charge of the contractor's employees has expressed this idea as related to dissipation following payday:[19] "'* * * a majority of cases occur following payday, and can be assumed to have dissipation as an etiological basis.'" However, there is no particular evidence, other than by casual observation, apparently, that there was an increase in dissipation following payday. Table 5 does show a marked increase in the incidence of compressed air illness

TABLE 5.—INCIDENCE OF COMPRESSED AIR ILLNESS ON QUEENS MIDTOWN TUNNEL, BY DAYS OF THE WEEK (FOR 20 TO 37.5 LB PER SQ IN. GAGE PRESSURES)

Day	Total decompressions	Number of cases	Cases per 100 decompressions
Sunday (watch shifts)	15,715	64	0.407
Monday	90,303	112	0.124
Tuesday	100,567	177	0.176
Wednesday	100,239	194	0.194
Thursday (payday)	100,312	200	0.199
Friday	101,891	203	0.199
Saturday	99,637	193	0.194

after exposure on Sunday, which was covered by small watch shifts with little work to do. On the basis of such working conditions, a minimum incidence of compressed air illness would be expected rather than double the average incidence, as actually experienced. Could this phenomenon be due to the fact that Sunday follows Saturday night so closely?

The New York State regulations, under which the Queens Midtown Tunnel was constructed, provide that

"Whenever the air pressure in a tunnel heading exceeds twenty-one pounds per sq. in. above atmospheric pressure, two air chambers shall

[19] "Engineering Problems Related to the Health of Workers in Compressed Air," by Ole Singstad, 6th Pacific Science Cong., Vol. VI, p. 82.

always be in use, except for such time as may be necessary when headings are being started from shafts; and whenever practical the pressure in the outer chamber shall not exceed one-half the pressure in the heading."

This condition is established by constructing the necessary additional bulkhead and is generally worked out in conjunction with the maintenance of a bulkhead within a specified maximum distance of the heading, generally about 1,000 ft. Fig. 11A shows the location of the bulkheads forming the outer air chambers in the two Manhattan headings of the Queens Midtown Tunnel, and Fig. 11D shows the locations on the Queens side. Using the single bulkhead and no outer chambers, 270,672 decompressions were made from a 22 to 37.5 lb per sq in. range of pressure with 486 cases of compressed air illness, an incidence of 0.179 per 100 decompressions. Using the additional bulkheads and outer chambers, 316,595 decompressions were made from a 22 to 37.5 lb per sq in. range of pressures with 646 cases of compressed air illness, an incidence of 0.204 per 100 decompressions. As far as controlling compressed air illness is concerned, this record shows no advantage in the use of the outer air chambers; in fact, the record for the single air chamber is better. The use of the outer chamber has been looked on as a great aid in the prevention of compressed air illness but the results on the Queens Midtown Tunnel, comparing the aforementioned large number of decompressions, indicate that this may not be the case. As the cost of installation, operation, and removal of a tunnel bulkhead is a substantial one, further study of this regulation is advised as it appears that the use of an additional bulkhead solely to satisfy the requirement for an outer air chamber may be an unwarranted expense. This consideration, of course, is entirely apart from the use of additional tunnel bulkheads when necessary to satisfy other safety requirements.

In further regard to decompression regulations, exception could well be taken to the requirement that the stage method of decompression be used. Although there is nothing in the record of compressed air work in the New York area, including that of the Queens Midtown Tunnel, definitely to prove this assertion, there is, on the other hand, nothing to prove the efficacy of the stage method of decompression as far as tunnel work is concerned. In fact, it well may be that the fast initial drop of pressure employed in this method of decompression actually induces the beginning of the formation of the air bubbles that are the cause of compressed air illness. The initial drop to half the gage pressure, as practiced on the Queens Midtown Tunnel, was often slower than the rate of 5 lb per min required by the regulations. Nevertheless, it is possible that an even slower initial rate of decompression might have given better results. This should be investigated on the next compressed air tunnel project. It should be remembered that the stage method of decompression was developed in connection with diving, where the diver is raised slowly to the surface and is subject, meantime, to the low temperatures of the surrounding water. The stage method is designed to achieve what may be considered a safe decompression in the minimum time; and therefore it is a desirable method in deep water diving, for which it was developed, where the effect of cold increases the diver's susceptibility to compressed air illness.

As tunnel workers decompress in a heated lock, a decompression method well may be used which sacrifices some small saving in time to greater safety.

It is interesting to note that, of the total of 1,148 cases of compressed air illness reported on the Queens Midtown Tunnel work, 194 or 16.8% occurred following the first of the two periods of daily exposure and that 954 cases or 83.2% occurred after the second period. Such a difference between the incidence of compressed air illness after the first and second periods of exposure may be explained in part by fatigue. The fact that some cases develop after the first period and are not reported until after the second (the men hoping to relieve themselves during the second exposure) may contribute slightly to the increased number of cases occurring after the second shift, but not to any great extent, as compressed air illness is painful and it is not likely many would wait several hours for relief.

In the lower pressures, especially up to 18 lb per sq in. and possibly higher, some consideration should be given to the use of one daily work shift of 6 hours, rather than two shifts of 3 hours. This modification would be in line with suggestions that have been made from time to time since the early days of the use of compressed air in construction, and the experience on the Queens Midtown Tunnel indicates that it might be practicable as there would be certain operational advantages without adverse effect on safety. This idea at least should receive further study.

ROBERT F. LEGGET,[20] ASSOC. M. AM. SOC. C. E.[20a]—The literature of civil engineering has been enriched by this notable account of a notable undertaking, an account enlivened by attention to many unusual details of design and construction. As was to be expected for such a public work in the vicinity of New York, due attention was paid to the necessity for preliminary subsurface exploration. The costs of this work as given in Table 2, amounting to only 0.2% of the total contract cost, provide further evidence of the economic value of such exploratory work. The successful performance of the Queens Midtown Tunnel makes any discussion of the main parts of the paper of academic interest only. There are some details, however, that the writer would like to see amplified, if only briefly, and he therefore ventures to comment upon them.

(*a*) Under the heading "Design of Structures: Lining for Shield-Driven River Tunnels," the author gives some details of the fastening of the bolts used for securing the tunnel liner plates and it is noted that "Actual bolt stresses * * * varied from intensities considerably less than 25,000 lb per sq in. to stresses approaching the yield point of the bolts." Would Mr. Singstad advise whether the ratchet wrenches used for bolt tightening were merely standard wrenches or, alternatively, wrenches designed to give a fixed maximum torque? Use of wrenches of the latter type is standard practice in certain branches of aircraft work, great ingenuity having been displayed in their design. It is to be hoped that one of the many minor by-products of the present wartime

[20] Asst. Prof., Dept. of Civ. Eng., The Univ. of Toronto, Toronto, Ont., Canada.

[20a] Received by the Secretary July 29, 1943.

industrial development will be the acceptance of such tools in civil engineering practice, particularly since bolts and turnbuckles form so important a part of many structures—being designed for definite initial stresses and yet all too often installed without reference to this requirement. The writer has in mind particularly the turnbuckles used for supporting bulkhead retaining walls as equivalent in importance to the bolts described in this paper.

(*b*) In dealing with the Manhattan plazas and approaches, the author states that one design for the plaza was not adopted since construction costs, for a thin invert slab and drainage system, were about the same as those for a very heavy slab design. This suggests that the drainage involved must have been unusually complicated. Could Mr. Singstad amplify this comparison of designs in order to make clear why drainage thus had to be neglected?

(*c*) Finally, in view of the "structural" use to which the clay in the river blanket was put, could Mr. Singstad give details of the type of clay used for the blanket, and also of the material referred to as "volcanic clay."

Corrections for *Transactions:* In March, 1943, *Proceedings*, page 374, Fig. 5(*b*), "Air Ducts Between Tunnels" refers to the left-hand section and "Air Ducts Top" refers to the right-hand section; page 384, line 10, change "contract No. 7" to "contracts Nos. 7 and 8"; page 384, line 12, change "these three" to "contracts Nos. 4, 7, and 8"; page 394, line 22, change "Fig. 11B" to "Fig. 11A"; and page 396, line 13, change "1,145" to "1,148."

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

DEVELOPMENT OF THE CHICAGO TYPE BASCULE BRIDGE

Discussion

BY MESSRS. CLINTON D. HANOVER, JR., AND ARMOUR T. GRANGER

CLINTON D. HANOVER, JR.,[8] ASSOC. M. AM. SOC. C. E.[8a]—As stated in the "Synopsis," this paper describes the development of a bridge type serving peculiar local requirements. Mr. Becker's treatment of his subject, in chronological form, is interesting as a record of the evolution of this one type of bridge and as a record of the experiences of one agency in the design and operation of movable bridges. Unquestionably, other agencies—such as states, other cities, and railroads, who design and operate movable bridges—have similar and parallel records of experience and development on various types of movable bridges.

The original sources of some developments in the art of movable bridge design might be hard to trace, since not only engineers employed by agencies owning the bridges, but also engineers employed by contractors, manufacturers, and consulting engineers, have from time to time added their share to the general development. Many of these improvements in design, especially improvements in the design of details, are reflected in the latest specifications prepared under the auspices of various engineering societies. These specifications are based on the experiences of many practicing engineers and apply to all types of movable bridges.

However, the choice of the general type of movable bridge is still a question for determination on the basis of conditions at the site and on the basis of the requirements of navigation and highway or railway traffic. For this reason, a full discussion of the advantages of any one type of movable bridge is of interest to engineers responsible for the selection of types for use at various locations.

NOTE.—This paper by Donald N. Becker, M. Am. Soc. C. E., was published in February, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1943, by Alonzo J. Hammond, Past-President and Hon. M. Am. Soc. C. E.

[8] Chf., Bureau of Bridge Design, Dept. of Public Works, New York, N. Y.

[8a] Received by the Secretary June 28, 1943.

Mr. Becker states that, with the development of the City of Chicago, the rivers were deepened and widened, providing channels varying in width from 100 ft to 250 ft. Table 1 gives among other things, the clear span and the width of the river channel for each of the Chicago-type bascule bridges on these rivers. It is interesting to note that, for many of the bridges, the clear span exceeds the width of river channel considerably. As an example, a few of these are as follows:

Location	Width of river channel, in feet	Clear span, in feet	Ratio, clear span to channel
West Division Street (Canal)	90	135	1.50
West Division Street (River)	100	155.67	1.56
North Western Avenue	121	184.58	1.52
North Halsted Street (Canal)	90	184.58	2.05
West Webster Avenue	120	161.25	1.34
West Belmont Avenue	75	161.25	2.15

It is the writer's understanding that in each of these cases the clear span exceeds the river channel because the bridge does not cross the channel at right angles but crosses at a considerable skew, and that in each case a normal bascule, or bascule without skew, was used.

As the leaves of a normal bascule bridge rotate on an axis which is normal to the center line of bridge and center line of roadway, it has been necessary at skew crossings to provide spans with a clear distance between piers considerably in excess of the clear width of channel. Also this condition has required that the large rectangular bascule piers be placed at an angle to the direction of the channel flow, thus disturbing the flow of the current as well as blocking off a greater width of the waterway to the flow of current. At such locations, in order to protect river traffic, it also has been necessary at times to provide independent fender systems of considerable cost extending along the lines of the real channel.

In 1940 the Department of Public Works of New York, N. Y., began studies for a new bascule bridge at Hamilton Avenue over the Gowanus Canal in Brooklyn. The intersection of Hamilton Avenue and the canal makes an angle of 56°. The old bridge at this location was a skew bascule bridge of the rolling lift or Scherzer type. The main girders of this bridge were framed normal to the channel lines, the axis of rotation being parallel to the channel. This resulted in the main girders being placed rather close together, thus limiting the available space for counterweights and machinery, and in large triangular parts of the roadway framing being cantilevered outside of the main girders. Although the clear channel width was only 46.5 ft, in order to obtain any semblance of substantial framing, a double-leaf bascule bridge was used. The bridge was built in 1905 and had proved to be structurally somewhat flexible, resulting in high maintenance costs.

In making studies for the new bridge several schemes were tried, among them a skew bridge, similar in arrangement to the original one, and a double-leaf bridge without skew. The skew design was not favored because of the experiences with the former bridge. A double-leaf bascule without skew would require a clear span parallel to the bridge approximately twice as great as the

width of the skew channel. The estimate of cost of the bridge without skew was so great that it, too, was not considered with favor.

On further study, the writer developed a new type of skew bascule bridge that has the advantages of a bascule without skew, without the disadvantages attended by the extra length of span. The forward leaf of the bridge is made to fit the skewed channel crossing precisely, and the rear leaf is built normal

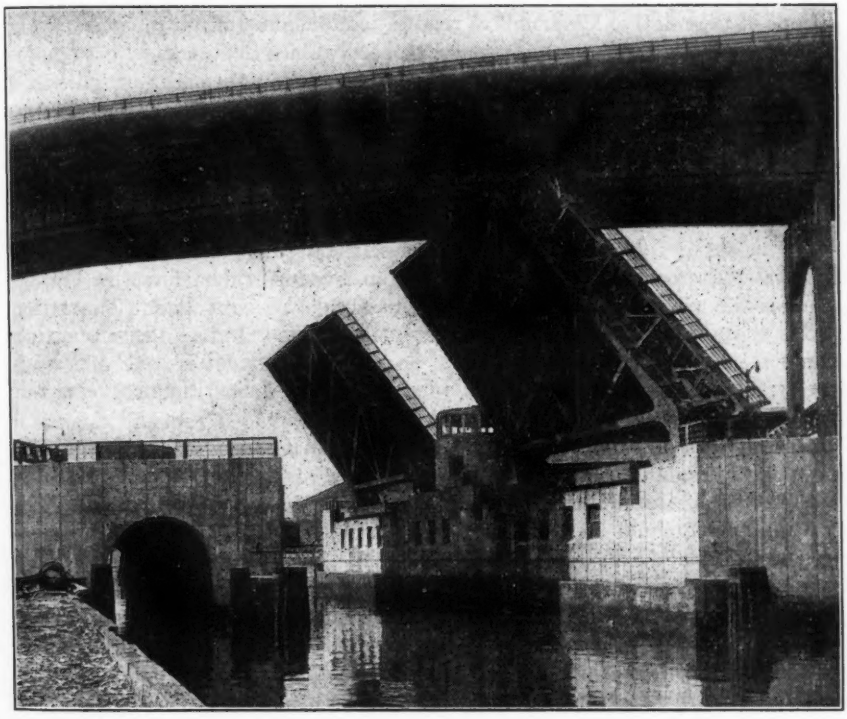

FIG. 20.—VIEW OF THE UNDERSIDE OF THE BASCULE LEAF IN OPEN POSITION

to the channel lines, thus making the forward leaf as short as possible and allowing the center line of trunnions to be parallel to and as near to the channel line as possible. This gives the shortest span from center line of trunnions to toe end of span that can be arranged. The main girders are spaced as far apart as may be desired, thus permitting complete and substantial bracing systems to be used, allowing ordinary concrete in the counterweights for counterbalancing, and providing maximum room for locating the operating machinery between the main girders. Since the piers are built in line with the channel, the channel flow is obstructed the minimum amount.

Although the center of gravity of the forward leaf is not midway between the trunnions or supports, the center of gravity of the counterweight can be midway between the trunnions, thus providing a counterweight of uniform section throughout its length. Fig. 20 is a view of the underside of the bascule

leaf in the open position, showing the framing of the main girders as well as that of the other structural members of the forward leaf. The adoption of this type of bridge allowed the use of a single-leaf bridge and resulted in a great saving in cost. The same scheme can be used in a double-leaf layout for wider skewed channels.

This experience in the City of New York may be of interest in other cases where skew crossings are involved. It is the writer's belief that natural conditions of the relative alinement of roadway and channel result in considerably more skew crossings than square ones, especially since the tendency is to maintain a straight alinement on highways to accommodate modern high speed traffic.

Mr. Becker states that in the design of the South Canal Street bridge some new ideas are incorporated, among them the use of open grating for the roadway pavement. This type of grating early was recognized as economical if used as a floor for movable bridges. The City of New York has nine bascule bridges with this type of decking installed, some of them older bridges on which the floor has been reconstructed. The Hamilton Avenue bridge shown in Fig. 20 is one of the new bridges on which this type of floor was installed. The writer agrees that its use saves weight not only in the floor but in the supporting members and in the counterweight. In addition, it reduces the operating forces required, as both friction and wind load are lessened, thus reducing the power and the weight of machinery required.

It is interesting to note that, on the South Canal Street bridge design, the use of an hydraulic drive is contemplated. In 1937 the writer made a study of this type of installation as it was recognized that speed control with the hydraulic drive could be very much more sensitive than with an alternating current electrical drive. However, at that time, the idea was rejected because the manufacturers of the various hydraulic drives available were not in a position to assist in the development of the automatic control necessary to duplicate the protection provided by limit switches in the standard electrical setup. Duplication of this protection required the redesign of the hydraulic units so that these automatic controls could be attached to them. Another factor against adoption of the hydraulic drive was that some systems worked on a high-pressure basis and others on a low-pressure basis, and, in case of emergency replacements being required, there necessarily would have to be full dependence upon only one or two manufacturers.

ARMOUR T. GRANGER,[9] M. AM. SOC. C. E.[9a]—All engineers engaged in the design of movable bridges should be interested in this timely and valuable paper. The comprehensive account of the development of the Chicago bascules from the crude structures of the past century to the excellent and artistic spans of the present reveals strikingly the tremendous advances which have been made in the design of bascule bridges during this period.

Pound for pound and dollar for dollar, the bascule probably presents more intricate problems than any other type of bridge structure. The successful solutions of many such problems are presented in the paper. Certainly the

[9] Prof. and Head, Dept. of Civ. Eng., Univ. of Tennessee, Knoxville, Tenn.
[9a] Received by the Secretary June 28, 1943.

engineers who have participated in the design and development of the Chicago bascules have contributed greatly to the science of designing movable spans.

The writer is strongly of the opinion that the simple trunnion bascule, with the counterweight beneath the floor, as exemplified by the Chicago bridges, is in general the most desirable of all bascule types, including those types of double-leaf bascules which require the span to function as a simple span or as anything other than a pair of shear-locked cantilevers when the leaves are closed.

Among the many interesting and difficult problems requiring solution in the design of a bascule are the following: Method of supporting the inside trunnion bearings, details of floor breaks, rail joints, provisions for trolley wires during movement of span, span locks, arrangement of counterweights, arrangement of operating machinery (frequently in limited space), adequate provision for proper machinery alinement, prevention of moving interferences, and many others. Some of these have been discussed in the paper. Limitations of space doubtless have precluded the consideration of others of interest and importance. It is hoped that in his closure the author may comment on more of these details.

It would be of interest, for example, to have some information relative to the development of the center locks for double-leaf spans. Various types of locks have been devised and used and some of them have not proved satisfactory. (This statement is general and is not made in reference to the Chicago bascules.) A thoroughly satisfactory lock should grip both leaves firmly, without play or clearance, should be arranged to engage and disengage readily, and should be so designed that no part of the lock can foul either leaf. The lock mechanism, of course, should be so interlocked with the operating machinery that power to operate the span cannot be available until the locks are opened fully. Any lock which involves vertical play or clearances with the lock in closed position is likely to cause objectionable bumping or impact when loads pass over the floor break.

The details of the center floor break are also important. Such breaks should be designed to insure smooth riding over the joint under all conditions. The overlapping tooth joint, with teeth tapered in plan and rounded at the ends on a vertical radius, can be arranged so as to be entirely satisfactory for the center roadway break of a double-leaf bascule.

Some discussion of the choice of the type of trussing used would be of interest. Warren trusses were employed for the West Lake Street bridge (Fig. 12), but apparently Pratt trusses were used for all the other bridges. It has been the opinion of the writer for many years that, except for shallow spans with deep members where secondary stresses may be of consequence, Warren trusses are preferable to Pratt trusses for practically all riveted bridge spans and the great popularity of the Warren truss in recent years evidences the fact that many engineers are of this opinion. A statement of the reasons why Pratt trusses were selected for the Chicago bascules should therefore be of value to the profession.

Another detail of much interest is the use of the live-load bearing or forward support in the front wall of the bascule pier. It would appear that the desirability of using such supports is somewhat open to question. In designing double-leaf, deck-girder, bascule spans the writer has investigated their use on

more than one occasion, but has omitted them. These supports will afford an appreciable saving in the weight of the forearm girders or trusses, but they have attendant disadvantages which tend to offset this. These disadvantages may be stated as follows:

(1) With the forward supports omitted, the double-leaf, trunnion bascule is statically indeterminate for live load to the first degree only, and the actual stresses can be determined quite closely. The introduction of a forward support in each leaf renders the span threefold indeterminate, and the computed stresses therefore are likely to be considerably more in error than if the forward supports are omitted. The fact that the inside trunnion support usually has a certain amount of flexibility contributes to the uncertainty as to exact stress distribution. Furthermore, since the relative simplicity of the trunnion bascule (as compared to other types) is one of its principal advantages, it seems rather a step in the wrong direction to detract from this simplicity.

(2) It is seldom, if ever, possible to make the forward support carry live-load reaction only. With full live-load covering both forearms between the forward supports, a part of the dead load on the trunnions will be transferred to the forward supports, and it is possible in some cases, where the live load is heavy, for the "live-load support" actually to be subjected to the full dead load on one trunnion in addition to the full live-load reaction.

The truth of this statement should be apparent from the following considerations:

With no live load on the span, the live-load bearing (forward support) and live-load anchor at the rear are adjusted presumably so that the parts of these details on the fixed and moving portions are just in contact. Now assume full symmetrical live load on both forearms—which condition will give zero shear at the center locks and negative moment for the full length of both leaves. As each leaf is held vertically at the forward bearing and rear anchor, that part of the span between these two points then will deflect upward. This upward deflection will reduce the dead load on the trunnion bearings and, unless the trunnion supports are quite flexible, enough dead load will be thrown off the bearings to the leaf to balance the moment of the live load on the leaf about the forward support, with a consequent increase in the reaction on the forward support. As the distance from the trunnion to the forward support is small compared to the length of the forearm, this increase in the reaction is large, and, with some span dimensions and live loads, it is possible to take the dead load completely off the trunnion bearings and transfer it to the forward support.

Such behavior of the span calls for heavy forward supports. Although the total resultant load on the foundation, of course, is in the same location with or without the forward support, the manner in which it reaches the foundation is quite different in the two cases. It is sometimes difficult to care locally for the heavy concentrations resulting from the forward support and to insure that they are distributed properly over the foundation.

Transference of at least part of the dead load back and forth from the trunnion to the forward support seems rather objectionable and appears to detract from the simplicity inherent in the trunnion bascule.

The writer should state again that his investigations of this matter have been made only on girder spans, but a similar line of reasoning should apply to truss spans.

Probably more ingenuity has been exercised in devising methods of supporting the inside trunnion bearing than in connection with any other bascule detail. Some very ingenious ways of accomplishing this on the Chicago bascules are described by the author. Another method, similar to the longitudinal, **S**-shaped girders used for many of the later bridges, consists of supporting the inside trunnion bearing on a very short girder hung from a horizontal longitudinal girder overhead which is supported by the front and rear walls of the pier. This arrangement can be used where the distance from crown of roadway to center line of trunnion permits.

Another good arrangement (patented) consists of supporting the inside bearing on a bracket cantilevered longitudinally from an inside trunnion column which rests on the floor of the pier. The column is so shaped in elevation as to fit into a notch in the counterweight to clear the span in the fully raised position.

It is desirable, but sometimes rather difficult, to provide approximately equal rigidity in the supports for inner and outer trunnion bearings. If there is much difference in the dead-load deflection of these supports, trouble may be encountered during and after erection.

The writer has had occasion to investigate the matter of transverse centering devices for the leaves of girder bascules and always reached the following conclusions:

(*a*) Transverse centering was not needed; and
(*b*) Any device that would deflect the leaf laterally by any appreciable amount would need to be so large and powerful as to be impracticable.

There are a number of spans with wide roadways, having interlocking tooth center breaks and no transverse centering devices, and so far as the writer is aware there has never been any transverse fouling on these spans.

The author has made a few comments relative to the comparative appearance of bascule and vertical lift spans, and a word on this subject may be in order. Having been connected with the design of many spans of both types, the writer is decidedly of the opinion that the bascule span is not necessarily superior in appearance to the vertical-lift span. Certainly many deck or railing-height bascules are more attractive than many vertical lifts, but in the eyes of many engineers nothing is more hideous than some types of through bascules with overhead counterweights. With thought and care the lift span can be given a very fine appearance. In the opinion of the writer, the Harlem River (Triborough) lift span in New York, N. Y., is not excelled in appearance by any movable bridge in the world. Certainly, however, the required horizontal and vertical clearances at Chicago, in general, are not particularly favorable for lift spans.

In conclusion, the writer would like to state clearly that no statement made in this discussion is intended to imply unfavorable criticism of any point in the design of the Chicago bridges. They are certainly a worthy monument to the city and the engineering profession, and the paper is a valuable addition to the literature of movable bridges.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SIMPLIFIED ANALYSIS OF SKEWED REINFORCED CONCRETE FRAMES AND ARCHES

Discussion

BY MESSRS. C. D. GEISLER, AND ARTHUR G. HAYDEN

C. D. GEISLER,[7] ESQ.[7a]—The subject of this paper long has been more or less of a nightmare to structural engineers. When confronted with the task of designing a skewed frame or arch, the designer has been obliged to wrestle with intricate mathematical equations.

The writer favors Mr. Hodges' simple method of procedure, in which first the frame is analyzed as a rectangular structure in computing the horizontal reaction H, and then corrections are made for the varying effects of the skew at the different points along the arch axis. It is surprising, however, that the torsional moments, which determine the amount of transverse reinforcement, have no effect in concentrating the dead load reactions and the live load reactions at the outer edges of the frame, thus increasing the main stresses at the obtuse corner.

Mr. Hodges' analysis of the main stresses would appear to agree with the theory advanced by Rankine and other early authorities who assumed that the stresses in a skewed arch were similar to those in a symmetrical right arch of span equal to the span on the skew.

The presence of certain concentrated stresses at the corners of a skewed barrel structure long has appeared evident from various model tests, and also from the results of tests conducted from 1922 to 1925 by the Bureau of Public Roads[8] on model concrete arches, with skews from 15° to 60°. Investigations of actual failures of concrete arches likewise have led to this conclusion. However, many concrete rigid frames of considerable skew have been constructed during the fifteen years since about 1928, apparently without added allowance for concentration of stress at the obtuse corners, and no reports have come to

NOTE.—This paper by Richard M. Hodges, M. Am. Soc. C. E., was published in May, 1943, *Proceedings*.

[7] Senior Highway Bridge Engr., Public Roads Administration, Federal Works Agency, Washington, D. C.

[7a] Received by the Secretary June 21, 1943.

[8] "Progress Report on Skew Arch Tests," by G. W. Davis, *Public Roads*, Bureau of Public Roads, U.S.D.A., Vol. 6, No. 9, November, 1929, p. 185.

the writer's attention of any indication of distress in the skewed barrel frames at or near these corners.

It is somewhat doubtful if extreme concentrations of reactions occur to the same degree in the rigid frame type of bridge as they apparently do in arches, since structures of the frame type generally are not founded on rock and any slight yielding of the soil under the loaded corners would cause the resultant of the reactions to shift toward the center of the abutment with a reduction rather than an increase in the critical stresses. On the other hand, a lateral movement of the abutments due to side thrust would tend again to increase the concentration of horizontal reactions at the obtuse corner.

Reports of failures in skewed barrel structures appear to be confined generally to true arches, which have a greater horizontal thrust, and depend for their stability on an unyielding foundation. A relatively thin section for the arch barrel above the springing lines will satisfy design requirements due to the absence of the high bending stresses which exist in a flat-top frame.

Since 1933 some twenty skewed concrete frames and arches (most of which have been constructed on National Park projects) have been designed under the writer's direction. These structures comprise a variety of barrel-type arches ranging from flat-top frames to circular and elliptical arches, with spans up to 114 ft and skews up to 45°. In these designs the final stresses were determined after calculating the eccentricity of reactions, based on the relative stiffness of elements radiating from the superimposed load to the abutments and involving the angle of skew, span length, and width. The frame or arch barrels were divided into longitudinal sections or strips usually from about 10 ft to 12 ft in skew width, and these extended without transverse construction joints from abutment to abutment. A 10-ft width was found to be about the minimum for practical construction operations. In the computations, the individual section was designed as an independent arch rib, and the stress analysis of the structure was based on the investigation of this rib section. The analysis involved the assumption that each section would deflect independently of the adjacent sections and that no load would be transferred across the longitudinal joints. Inasmuch as the concentration of reactions at the obtuse corners depends on the effective width of the structure, if the barrel is constructed in independent longitudinal sections, each section must transmit its own load to the abutments in much the same way as a timber roadway plank laid diagonally carries its load to the stringers. However, in order to provide lateral distribution for heavy concentrated live loads and to prevent deflections which might break up the roadway pavement in cases where a thin roadway surfacing lay directly on the frame, the sections were keyed together for a length of a few feet at the crown of the arch. Furthermore, to tie the structure transversely, particularly in structures retaining a spandrel fill, transverse reinforcement extended over the full width of a structure, crossing the joints between successive sections. To what extent the keys at the crown and the transverse reinforcement might act to upset the assumption of independent deflection of each strip is questionable, of course, but it is doubtful that the small keying action so furnished would be sufficient to cause deflections in the

successive concrete sections which could produce serious overstressing in the outer edges.

The usual computations for reactions in a statically indeterminate reinforced concrete structure are based on relative deflections in which the concrete is assumed to be a homogeneous material in both tension and compression. This would be more nearly correct in an arch than in a frame, since, in a true arch, the line of resultant thrust departs but little from the middle third when only small tension stresses occur. In the computations for deflection of a reinforced concrete barrel frame in a diagonal direction, or along an element not parallel with the reinforcement on which the tension steel can act only directly, considerable error may be involved. An element crossing the reinforcing steel diagonally would likely develop tension cracks under flexural stress, resulting in a reduction in its effective moment of inertia. Its capacity as a beam for transmitting any considerable amount of load to the obtuse corners then would be decreased.

It is quite conceivable that in many existing skewed barrel structures local concentration of forces, if this occurs, could produce overstresses of as much as 50% without giving any indication of distress, particularly in a structure that otherwise is designed conservatively for moments and side thrusts.

Tentative conclusions drawn in 1925 at the completion of the foregoing tests made on models of true arches by the Bureau of Public Roads were as follows:

1. The vertical reactions varied uniformly from a maximum at the obtuse angle of the arch to a minimum at the acute angle.
2. The increase of the vertical reaction at the obtuse angle of the arch varies directly as the angle of skew.
3. The resultant pressures of both the horizontal and vertical components move toward the obtuse angle of the arch with an increase in the skew.
4. The direction of the side thrust is toward the face of the arch at the acute angle and the intensity increases with the degree of skew.

The foregoing conclusions could be interpreted to indicate that the reactions from a superimposed load on a span have a tendency to follow the shortest distance to the abutments. In a beam and girder bridge, the reactions are determined readily by the relative stiffness of the supporting members. In a slab or barrel arch structure, however, a close determination of the reaction distribution is more involved. Fortunately, in the latter structures a concentration of load is seldom serious.

ARTHUR G. HAYDEN,[9] M. AM. SOC. C. E.[9a]—The development of analysis for skewed arches and rigid frame bridges has been a subject of intimate interest to the writer from the beginning. Therefore, he can appreciate the value of Mr. Hodges' contribution to the literature on the subject.

Historical Perspective.—In 1920 the writer was assigned the duty of revising plans for an arch bridge on a very heavy skew (more than 50°) and sensed that something was wrong with the design. Undoubtedly, if the bridge had been

[9] Cons. Civ. Engr., New York, N. Y.; formerly Designing Engr., Westchester County Park Comm.
[9a] Received by the Secretary August 2, 1943.

built according to the original plans, the ends of the arch ring proper would have slipped along the very smooth skewbacks of the abutments which had already been built, and a collapse would have followed. There was comparatively little steel reinforcement projecting from the abutments for anchoring the arch ring thereto. A thorough search was made in technical literature, and it was discovered that several attempts had been made by engineer-mathematicians to solve the problem of the skew arch; but all of the analyses discovered were fallacious. All that the writer could do was to saw-tooth the skewbacks of the abutments already built to prevent slippage of the arch ring when built thereon. An accurate analysis of the stresses throughout such a structure seemed impossible, and the design of several skewed bridges on the parkways of Westchester County was in prospect. Happily, a paper by Professor Rathbun appeared in the February, 1924, *Proceedings* of the Society, giving a solution of the fixed skew arch problem.[3] This paper has proved to be epoch-making in the field of structural analysis. Unfortunately, the validity of Professor Rathbun's theory was questioned at first, and a practical test of the theory was authorized by the Society and assigned to a Special Committee on Concrete and Reinforced Concrete Arches.[10] The late George E. Beggs, M. Am. Soc. C. E., of Princeton University in Princeton, N. J., was engaged to make a deformeter analysis on models of skewed arches and Professor Rathbun, then of the South Dakota School of Mines, in Rapid City, was engaged to make mathematical analyses for structures of the same proportions. Professors Beggs and Rathbun were not in communication with each other during the progress of the work, but, when both parties had completed their assignments, results were compared and were found to be in very close agreement. This was accepted as a verification of Professor Rathbun's theory.

In 1925 Professor Rathbun was engaged to extend his theory of the skewed fixed arch to skewed rigid frame bridges, because the Bronx Parkway Commission and its successor, the Westchester County Park Commission, was faced with the design of a number of bridges of this type for grade separations on Westchester County parkways. Professor Rathbun's work was accepted, but, in order to make it more useful in the designing room, an adaptation was made.[11] Apparently even this "simplification," involving as it does the derivation and solution of four simultaneous equations, was not suitable for use by the average designer, and, because of this, many bridges have been built on poor alinement, or unsatisfactory types of structure have been used in order to circumvent the analysis of a skewed arch or frame bridge.

Early in 1942 Mr. Hodges and the writer were engaged on the analysis of a heavily skewed rigid frame bridge for the highway network near the new War Department Building at Washington, D. C. The writer saw that Mr. Hodges was near the goal of a genuine simplification, and urged him to prepare the paper now under discussion. The value of the work can be appreciated when one considers that the solution of four simultaneous equations is now reduced

[3] "Analysis of the Stresses in the Ring of a Concrete Skew Arch," by J. Charles Rathbun, *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

[10] "Concrete and Reinforced Concrete Arches," Final Rept. of the Special Committee, *ibid.*, Vol. 100 (1935), pp. 1551–1573.

[11] "The Rigid-Frame Bridge," by A. G. Hayden, John Wiley & Sons, Inc., 1940, 2d Ed., Chapter X.

to the solution of two. Moreover, the author makes the subject much more lucid.

Observations and Suggestions.—Mr. Hodges assumes, to begin with, a "residual" or "transformed" system in which all redundant reactions have been removed except the horizontal thrust as calculated perpendicular to the abutments. He then analyzes the structure on the square span and derives the remaining redundant reactions. It appears that he could as well have taken a residual system in which all redundant reactions have been removed except the horizontal thrust as calculated parallel with the skew. Stresses would be analyzed on the skew span and the remaining redundants derived. Subsequent transformation of stresses in the square direction to stresses in the skew direction would be avoided. Although the same fundamental principles would be involved, the redundants would be derived differently. The final total reactions, however, would be the same. It has been shown that, in general, any one of a number of "transformed" or "residual" systems may be selected as a basis for deriving redundant reactions, the number of possible residual systems depending upon the number of redundants to be calculated.[12] Some of these systems involve much less mathematical work in solution than others. So, in the case of the skewed frame bridge, one of the residual systems may involve less work in solving for the redundant reactions due to the effect of skew than the other system. Such a comparison might yield valuable results.

A common misconception in regard to skewed arches or frame bridges is that the effect of skew on the stresses can be reduced by the use of T-beam construction. Unless the floor slab is distinctly separated from the ribs so that the two elements do not act as T-beams, skew stresses will exist in the structure to full extent and should be accounted for in design. As a matter of fact, Professor Rathbun has warned[13] that the problem is made even more complicated by the use of T-beam construction. If all the ribs did act independently, the eccentric loading from earth pressure at the ends of the bridge would become troublesome.

[12] "The Rigid-Frame Bridge," by A. G. Hayden, John Wiley & Sons, Inc., 1940, 2d Ed., Chapter IX.
[13] *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 1471.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

PHYSICAL PROPERTIES THAT AFFECT THE BEHAVIOR OF STRUCTURAL MEMBERS

Discussion

BY MESSRS. R. L. MOORE, LEON S. MOISSEIFF, AND FRED L. PLUMMER

R. L. MOORE,[26] ASSOC. M. AM. SOC. C. E.[26a]—An excellent general discussion of the need for ductility in structural steels is presented in this paper. The author's ideas also are applicable to other materials than steel. Although the functions of ductility generally are recognized, it is difficult to place a quantitative value on the importance of this property or to establish limits within which it may be varied without sacrificing structural safety. The author's statement in the "Synopsis" implying a definite relation between prescribed physical properties, including ductility, and the behavior of structural members, seems to the writer to require some elaboration.

As long as the structural engineer was concerned principally with the use of carbon steel, the question of ductility was not important. The present trend toward the use of less ductile alloy steels and nonferrous metals for special uses, however, particularly where weight saving is an important consideration, makes it necessary to examine ductility specifications more critically. The question of greatest interest is not, "How is ductility useful?" but rather, "How much ductility is needed?"

Ductility generally is expressed in terms of percentage elongation and reduction in area as determined from standard tensile tests. Ratios of tensile yield to ultimate strength also give a measure of ductility. Specifications for certain alloy steels limit this ratio to a maximum of 0.70,[27] whereas for carbon steels it usually ranges from 0.50 to 0.60. It should be emphasized that all materials do not exhibit the same characteristics of ductility. Among the heat-treated

NOTE.—This paper by Wilbur M. Wilson, M. Am. Soc. C. E., was published in December, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1943, by Jonathan Jones, M. Am. Soc. C. E.; May, 1943, by A. B. Kinzel, Esq.; and June, 1943, by Messrs. Almon H. Fuller, and C. F. Goodrich.

[26] Research Structural Engr., Aluminum Research Laboratories, Aluminum Co. of America, New Kensington, Pa.

[26a] Received by the Secretary June 3, 1943.

[27] "Specifications for Steel Railway Bridges," A.R.E.A., Pt. 3, Alloy Steels, 1938.

aluminum alloys, for example, alloy 61S–T, with a ratio of tensile yield to ultimate strength of 0.85 and an elongation in 2 in. of 14%, may show better ductility as far as forming operations are concerned than alloy 17S–T, with a ratio of tensile yield to ultimate strength of 0.65 and an elongation of 20%. If reduction in area is used as the measure of ductility, however, rather than the ratio of tensile yield to ultimate strength, or elongation, a more consistent indication of the relative forming characteristics of these particular aluminum alloys is obtained.

Carbon steel apparently has such a margin of ductility beyond that required for most structural uses that there has been little need to attempt to establish a definite relation between this property and the behavior of structural members. Observations on the behavior of actual structures are of little help in this respect since the latter are not designed to function in a stress range where ductility is a significant factor. Few structural tests appear to have been made in which the effect of ductility was one of the principal variables considered. It has been shown by static tests on riveted joints of steel and aluminum alloys[28] that differences in elongations ranging from a minimum of 11% to a maximum of 39% (measured over gage lengths of four diameters) had no apparent effect upon structural behavior. The joint strengths were within a few per cent of predicted values, indicating that the needs for ductility to level off stress concentrations and to effect a uniform distribution of load between rivets were amply met.

The author points out that the welding of structural steels may produce stress concentrations that are detrimental to the fatigue strength of a member. No mention is made of the effect of welding upon ductility. Tests by Professor Wilson, with Messrs. Bruckner and McCrackin,[15] as reported in 1942, have shown, however, that elongation values for butt welded tensile specimens of low-alloy steels may be as low as 5% in 8 in., when fractures occur in the welds. The effect of this "local brittleness" upon shock resistance and fatigue strength would appear to be an important consideration, although the increasing use of welded steel construction might be interpreted to mean that this reduction in ductility is not a critical factor.

Experience is needed, of course, to indicate to what extent ductility may be utilized in a structural material. The civil engineer, accustomed to thinking in terms of the ductility of carbon steel, will naturally have a different conception of this property, and of minimum structural requirements, than the aeronautical engineer, whose principal structural materials are alloy steels and nonferrous metals. The successful structural use, in aircraft, of metals having elongations of 10% to 14% in 2 in. (which is appreciably less than those generally prescribed for other types of structures) emphasizes the need for a better understanding of the functions of ductility with respect to structural behavior.

---

[28] "Riveted and Pin-Connected Joints of Steel and Aluminum Alloys," by Leon S. Moisseiff, E. C. Hartmann, and R. L. Moore, *Proceedings*, Am. Soc. C. E., January, 1943, p. 3.

[15] *Bulletin No. 337*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1942.

LEON S. MOISSEIFF,[29] M. AM. SOC. C. E.[29a]—The experience that engineers and metallurgists have had with structural members has brought to their minds in recent years an increasingly important problem. Professor Wilson deals with the information essential to this problem. He asks what physical properties of metals are desirable and may be required to assure the best behavior of structural members. The problem is not an outcome of theoretical considerations; it is a purely practical one. Based on his wide experience, Professor Wilson poses the problem on observations, engineering practice, and tests.

In the "Synopsis" of the paper the statement is made that "It is reasonable to infer that, if certain physical properties are prescribed, there must be some quite definite relation between these properties and the behavior of structural members." This suggests that the author doubts the correctness of the inference—and so do many engineers with him. It has been found in many cases that the physical properties prescribed in widely adopted specifications are insufficient to assure the desired behavior of structural members. As stated by Professor Wilson, many years of experience in fabricating and using structural carbon steel have proved that these specifications and the prescribed tests for the acceptance of the material have served well the requirements of the structures in operation. In fact, it should be emphasized that the specified physical properties of structural carbon steel and the acceptance tests for them have been developed by engineers during these many years of experience by closely following the behavior of the material, constituent members, and structures in use. Also, it should be kept in mind that commercial acceptance tests are limited by practical considerations. From the practical side it is important that such tests should be relatively simple and should consume little time. Acceptance tests have been standardized for fair comparisons and definite results and also to attain speed in testing.

Engineers early recognized that the strength of the material is the first requisite. The behavior of the material under action of load or strain also soon attracted the attention of the practical master mechanic—some kind of a bend test was used for metals long before scientific testing. In developing the standard tension test for strength, the phenomenon of ductility attracted attention and was made part of the tension test, largely because it was concomitant with it. (This is available with the tension test and consumes little additional time.) The observation and recording of the reduction of area also fall in line with the tension test. Should additional physical properties be specified, they will require separate tests which may be more time consuming.

It was very fortunate that, in the transition from wrought iron to steel, the structural carbon steel of the present time was developed. Its wide use for structures is largely due to its dependability and workability. Most of the latter is due to its high ductility. The high percentage of elongation of structural carbon steel is a result of its manufacture; it was not aimed for especially. Low-alloy and high-alloy steels, as well as light-weight metals, however, do not show such high elongation as structural carbon steel. This has brought up one of the questions of the new problem: How high an elongation

[29] Cons. Engr., New York, N. Y.

[29a] Received by the Secretary June 18, 1943.

does a metal require to meet the requirements of modern fabrication and erection of structures? The author discusses at length the several factors which demand from the metal an elongation through ductility that surpasses the elastic elongation. There are indications of a quantitative evaluation of these factors in the paper. It is highly desirable, however, to establish experimentally the percentage of elongation required in a given metal to meet, safely, the possible plastic deformations that may occur during fabrication, erection, and operation.

To study the effect of higher and lower ductility of metals and the effect of higher and lower ratios of yield strength to ultimate strength and the effect of the moduli of elasticity on distribution of stress, a series of tests of riveted and pin-connected joints was made.[28] Five metals—two steels and three aluminum alloys—having a fairly wide range of ductilities and ratios of yield strength to ultimate strength were included. All the joints tested behaved in a manner consistent with the basic assumptions of design and with the properties of the materials. Furthermore, the tests showed that the plastic yielding necessary to effect a uniform distribution of load among the rivets must be relatively small and that differences in ratios of yield strength to ultimate strength, elongation values, and moduli of elasticity were not significant factors as far as static ultimate strengths were concerned.

In these tests it was shown that the plastic yielding necessary to attain a uniform distribution of load in the riveted joints was relatively small. Tests on various structural parts in which stress-raising is expected may furnish information on the percentage of ductility required for practical purposes.

In discussing physical properties that affect the behavior of structural members, the writer thinks it is appropriate to mention some desirable physical properties which metallurgists may strive for.

One such property is the physical durability of the metal, its resistance to corrosion. In recent years a fraction of 1% of copper has been introduced to the chemical constituents in structural steels as protection against rust. There probably are other corrosion retarding agents which could be utilized economically. The assurance of a very low rusting quality would allow engineers to improve the design of structural members.

Another property of metals which is all important in the behavior of structures is their modulus of elasticity. It is amazing how the numerical value of the modulus remains practically unchanged for a given metal. It appears to be independent of the other physical properties and of the process of manufacture. Thus, rolled structural steel and high-strength stainless steel will show practically the same modulus as the cold-drawn bridge wire, which is nearly four times as strong as structural steel. Various grades of aluminum and duralumin have the same aluminum modulus; and other metals also have a specific value. This phenomenon is so striking that it arouses the curiosity of engineers. The satisfaction of this curiosity and raising the modulus of some metals would result in tremendous advantages of structural and economical value. These facts call for research of the highest kind. Whether the modulus of elasticity results from the forms of the crystals and their pattern in the metal and whether

these could be modified by controlled processes affecting their formation is a problem of transcendental interest. Structural engineers need not be told that successful research in these problems will result in substantial gains.

FRED L. PLUMMER,[30] M. AM. SOC. C. E.[30a]—The importance of the ductility of structural steels is emphasized in this paper as necessary in order to avoid possible failures due to: Fabrication and erection operations; stress concentrations caused by inaccurate fabrication, irregularities, welding heat or forced deformations; or excessive stresses caused by shock loads. J. A. Van den Broek and Hardy Cross, Members, Am. Soc. C. E., and others also have discussed the capacity of ductile metals to undergo relatively large deformations when stressed to the yield point, thereby modifying the distribution of stresses and possibly increasing the safe load-carrying capacity of the structure.

The ductility, to which these authors refer, is measured usually in connection with tension tests in which uniaxial stress is applied at room temperature and in small increments separated by definite time intervals or at a relatively slow but uniform rate. Thus, the reported values do not reflect the possible effect of temperature, rate of propagation of stress, or biaxial or triaxial stress distributions. Service failures of steel structures almost invariably show none of the deformations which are characteristic of failures in the tension tests. This indicates that ductility has failed to perform the functions suggested by the author, at least in these specific cases.

Professor Wilson refers to the structural steel covered by specification A7.[2] Many structural shapes are rolled from steel meeting this specification. Steel plates used for structural work often are selected to meet specifications A10 or A70. Such steel plates, ordered from different manufacturers and meeting all requirements of these specifications, may consist of "rimmed steel," "killed steel," or "semi-killed steel." However, metallurgists state that the resistance of these steels at low temperatures to impact forces and their susceptibility to age embrittlement following plastic deformation may be quite different.

Steel plate structures frequently consist of plates that have been bent or dished into curved shapes by rolling or forming. Thus such plates are subjected to more "cold working" than most of the structural shapes used in bridges and buildings although these members sometimes receive similar treatment. In steel plate structures the principal elements of the structure are subjected frequently to biaxial stresses of approximately equal intensities whereas the principal members of bridge and building structures normally resist only uniaxial stresses.

The writer would appreciate a statement from the author indicating whether he considers safe the use of steel plates meeting the requirements of specification A70 in a code-designed structure: (1) For which the plates will be formed cold to a dished shape (age embrittlement following plastic deformation); (2) in which biaxial stress conditions will exist throughout most of the structure (partial loss of effective ductility); (3) which will be subjected to atmospheric

[30] Chf. Research Engr., Hammond Iron Works, Warren, Pa.

[30a] Received by the Secretary June 30, 1943.

[2] Specifications, American Society for Testing Materials, Designation, ASTM—A7.

temperature variations in an area when for short periods the temperature will fall below 0° F (lower resistance to impact forces); (4) of which the various plates will be assembled by carefully controlled welding processes (stress concentrations due to welding heat); and (5) which, during such a period of low temperature, might be subjected to an impact force while resisting the normal design loads (speed-of-stress propagation). If the author does not consider the specifications adequate, in what way would he suggest that they be modified so as to insure the production of a steel that would meet the foregoing requirements adequately?

Structural engineers are widening continuously the scope of their activities. This applies to the types of structures which they design and to the kinds of metals to be used in such structures. Not many years ago the typical structural steel designer was concerned primarily with the use of one material, one method of fabrication, and the proper proportioning of members subjected to bending forces or to a direct thrust or pull, together with relatively simple "connections." Now he may choose from a wide variety of alloy steels and nonferrous metals in proportioning the parts of complicated continuous structures, including parts such as "skin structures" whose strength depends upon their elastic stability. He may connect the parts of his structure with rivets or by welding. It is only natural, but perhaps fatal, to try to apply old standards, tried procedures, and tested ideas to new materials used in new types of structures. Professor Wilson serves all engineers in outlining some of the problems and indicating a few about which additional information is needed.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

DEWATERING, INCINERATION, AND USE OF SEWAGE SLUDGE
A SYMPOSIUM

Discussion

BY MESSRS. LLOYD M. JOHNSON, AND ISADOR W. MENDELSOHN

LLOYD M. JOHNSON,[10] ESQ.[10a]—A wealth of information, covering the development and design of a system for dewatering and drying activated sewage sludge, is presented in this Symposium. Fundamental principles are presented in a logical and understandable manner by Messrs. Dundas and McLaughlin. This paper leads the sanitary engineer through the hitherto foreign fields of combustion, in easy stages. Therefore, it is a timely contribution to the development of a new art in sewage disposal—namely, drying for fertilizer purposes, or combined drying and incineration.

Three developments discussed by Messrs. Dundas and McLaughlin should be emphasized: Odor control, fertilizer production, and power generation.

Odor Control.—The control of odors is by high temperature deodorization, which consists of raising the temperature of all the gases and vapors in contact with the sludge during the drying period to at least 1,300° F. There is no argument concerning the necessity for high temperature deodorization in locations where odors would create a public nuisance. This has been recognized for decades in garbage incineration. In consequence, garbage incinerators must operate at deodorizing temperatures of 1,300° F and greater. Where odor control is required, high temperature deodorization is the most economical method. No costly chemicals and no special apparatus are required. High temperature deodorization is an integral part of the sludge-drying system.

Fertilizer Production.—The flash-drying system used by The Sanitary District of Chicago is well suited for fertilizer production. Furthermore, the system has great flexibility because the dried sludge can be incinerated whenever the fertilizer market is unfavorable. Before the sludge-drying and incineration

NOTE.—This Symposium was published in January, 1943, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: February, 1943, by A. L. Genter, M. Am. Soc. C. E.; and June, 1943, by Willem Rudolfs, M. Am. Soc. C. E.

[10] Commissioner of Streets and Electricity, Chicago, Ill.; formerly Engr. of Maintenance and Operation, San. Dist. of Chicago.

[10a] Received by the Secretary June 1, 1943.

system was developed, the end product of a sewage treatment plant was sludge-bed cake. This cake varied in moisture content from 20% to 60%. Thus, after passing through a highly developed and refined treatment process, the city was still confronted with the problem of disposing of huge quantities of sludge-bed cake. With the application of flash drying and incineration, the end product became either a highly desirable and salable fertilizer or a sterile ash well suited for fill. Every sludge, no matter how low in nitrogen content, has some value as a fertilizer and soil conditioner. Experience has shown that a ready market exists, and the sales value will materially help to compensate for the cost of treatment. On the other hand, it is necessary for the city to pay for the trucking and disposal of ash. Thus, as a general rule, provision should be made for fertilizer production in preference to incineration. For very large operations such as those at Chicago, the quantity of ash also becomes a problem. It must be remembered that dried sludge is 50% ash, and it is sometimes difficult to find sufficient low ground for disposal of this ash over a long period of time. Furthermore, ash is highly abrasive and requires considerable maintenance on any ash handling system.

The desirability of fertilizer production is by no means confined to the large cities. As a matter of interest, fertilizer is successfully produced and marketed in communities of 5,000 population.

Power Generation.—The operations at Chicago are unique in that power generation by steam is practiced. It is quite customary for power to be generated by gas engines fired with digester gas. The size of the operations at Chicago make power generation by steam a most economical solution. This process may be applied to much smaller communities if public utility rates are particularly unfavorable or if the location of the plant makes public utility power impractical.

ISADOR W. MENDELSOHN,[11] M. AM. SOC. C. E.[11a]—This Symposium of five selected papers affords an opportunity for deserved consideration of one of the major problems of sewage disposal. Presentation in such grouping makes for clearer comparison and more extensive analysis of data evolved in plant operation at Buffalo (B), Washington (W), Minneapolis-St. Paul (M), and Chicago (C) regarding sewage sludge dewatering, incineration, and final use. The Symposium also presents certain limitations which merit attention in rounding out discussion of sewage sludge disposal.

To the writer, one of the most significant features of the Symposium is the exceedingly large number of tests performed at the plants to develop satisfactory and economical operation in disposal of sludge. By no means do these include all experimental work on sludge disposal at these plants. For ready comparison these researches and results are grouped as follows.

Concentration of Sludge.—

(B) Sludge concentration tanks have proved their value by delivering sludge of reasonably uniform characteristics for filtration and burning, absorption of irregularities in quantity of raw sludge obtained, provision of a

[11] Capt., Corps of Engrs., U. S. Army, Asst. Post Engr., Camp Hood, Tex.

[11a] Received by the Secretary June 1, 1943.

6-day operating week in dewatering and incineration, and provision of gas as fuel for boilers and auxiliary heat for incinerators.

(M) Concentration of sludge by weighing a known quantity pumped from the sedimentation tanks at frequent intervals, by soundings and determinations of moisture content of sludge in tanks, and by withdrawals of supernatant liquor has increased the total solids of the sludge from 8.11% in raw sludge to 9.35% in concentrated sludge.

(C) Concentration of sludge has reduced the quantity of water to be removed by the filter and provided a filter feed of uniform consistency—a vital factor in successful filter operation.

Conditioning of Sludge.—

(B) A mixing tank with no baffles and with proper mixing-paddle speeds and a short detention period gave best results. In the application of chemicals, equipment has been developed to provide easy manual control in preference to automatic devices, such as feeding ferric chloride solution through a perforated rubber-pipe arrangement so that the number of small streams of solution is an indicator of the rate of feed.

(W) In the operation of elutriation mixing tanks, fine sand settling out in the tanks has been removed gradually to the vacuum filters with the sludge by connecting the tank inlet and outlet with a 12-in. pipe. To reduce sludge pump clogging and interference with continuous dewatering, the sludge pumps elevating the digested, elutriated sludge from the elutriation tanks to the mixing tanks were provided with 3-in. instead of 2-in. impeller passages and a maximum speed of 1,140 rpm. To overcome sagging of pipe lines between supports, steel pipe with hard-rubber lining was substituted for hard rubber ferric chloride pipe lines.

(M) Better results were obtained by increasing the number of points through which air is admitted to the sludge for mixing and by decreasing pressure and violence of mixing. Improved mixing resulted from changing the path of the sludge through the tanks with over and under baffles to a sinuous path both vertically and horizontally, and reducing the detention period in conditioning tanks from 13 min in 1938 to 7 or 8 min at the close of 1941. Installation of riffle boards for both lime and ferric chloride instead of adding chemicals to the tanks through one or two pipes provided better dispersion of chemicals. Ferric sulfate and lime and ferric chloride alone can be used satisfactorily as substitutes in emergencies, but they are not as economical by about 25% as ferric chloride and lime.

(C) In conditioning sludge for many years, ferric chloride alone has been found the most effective and most economical reagent available for conditioning activated sludge. With other sludges, combinations of ferric chloride and lime and in some cases lime alone are efficacious; chlorinated copperas and ferric sulfate are probably less effective. Special proportion devices for continuous conditioning and frequent checking of sludge to determine the amount of chemical have established better dosage control of ferric chloride, which has been found to be the most expensive item of filtration cost in handling activated sludge. The amount of ferric chloride required may be related to the volatile content of the activated sludge; ferric chloride requirements for conditioning

activated sludge of The Sanitary District of Chicago vary between 4% and 7% by weight of dry cake solids, and resultant cake moistures vary between 78% and 85%.

Operation of Vacuum Filters.—

(B) Canton flannel filter cloth has proved most satisfactory in operation, cost, and life of cloth; and ordinary sash cord has proved a satisfactory substitute for bronze strips in the application of the cloth to the filter drum. Experiments are in progress with a screw feeding device and a surge bin above the screw, to receive the sludge cake from the filter and discharge it uniformly to the first conveyer belt. Formation of lime scale on the wire screen on the surface of the filter drum was removed successfully with the application of muriatic acid in the filter tub; this acid also has been used successfully in reducing scale accumulations in filtrate lines and in the supernatant line receiving the discharge from the filtrate pumps.

(W) Washing filter cloths in a 1% solution of oxalic acid for a 24-hr exposure after certain period of use extended their life 30%; application of a 100-ppm solution of copper sulfate every Saturday and Sunday by rotating the filter cloth in the solution in the filter pan to permit saturation overcame filter-cloth disintegration by fungus growths in the regular filter shutdown from Friday afternoon to Monday morning.

(M) Eight-ounce Canton flannel filter cloth was found to be most satisfactory at this plant. The difficulty of obtaining this cloth during wartime necessitated additional studies leading to the possible use of a new cloth type. The economic life of filter cloths has been restricted to 350 hr; tests made on a plant scale to demonstrate the effect of cloth age on filter yields show that at 500 hr the yield of a filter is approximately 10% to 15% less than that of a new cloth, other conditions being constant, with 4-hr intervals between washings. An appreciable improvement is effected by proper washing, as noted by rearranging the washer sprays to impinge along the filter drums almost at right angles to the drums, instead of tangentially. It appears to be economical to give each cloth an acid bath after 350 hr of service (using 1 to 2 carboys of hydrochloric acid in 600 gal of water), thereby reducing the binding and increasing the economic cloth life to approximately 500 or 600 hr. (This expedient provides an inexpensive means of removing calcium carbonate from the screens and splines of the filters, which formerly became so incrusted with matter as to interfere with the free passage of water through the filter.) Especially is it economical in chemicals to wash filters frequently, the periods between washings having been reduced from 24 hr to 4 hr. Frequent filter washings led to the development of a method of washing filters while in service by installing the wash-water pipe and nozzles (instead of its usual location on the far side of the filter from the take-off plate) immediately below the top edge of the take-off plate, so that the washing operation is done in the section of the drum between the take-off and the submergence in the sludge, which section is not utilized in the usual installation. A trough suspended immediately above the sludge level returns the spent wash water to the overflow box on the filter. Besides maintaining a clean filter medium, this washing procedure in effect adds one filter. For each size of installation and type of sludge, it is possible to determine the economic

range of speeds at which filters ordinarily should be operated. The filter rate has a marked effect on the economics of filtration; instead of expressing the filter rate as pounds per square foot of installed filter capacity per hour, the index of pounds per square foot is favored to include varying filter speeds. Uniform feed is desirable where incineration or drying is the practice, and for this a spare filter unit is desirable in addition to regular operating filters for use during washing of regular filters.

(C) A long-nap woolen blanket is most satisfactory and economical for filtering activated sludge conditioned with ferric chloride. The life of a blanket is approximately 3½ months; the life of binding wire is about the same as the filter cloth. Use of woven wire for cloth backing has not proved satisfactory; wood backing may prove more satisfactory and economical than wire cloth.

Operation of Incinerators.—

(B) Mechanical developments at Buffalo include replacement of oil by gas exclusively as an auxiliary fuel, resulting in the disappearance of scale on preheaters caused by oil fuel, removal of soot blowers as unnecessary, and abrasion on heater surfaces making necessary replacement of some parts. Abrasion in pipe lines and ducts has been solved by use of concrete wear backs at bends; lower speeds in the flash-drier cage will have increased its life to about 5 months; adjustments in the type of assembly have resulted in easy replacement of worn parts and reuse of portions of these parts; moving the induced draft fan to the clean air side of the drying system cyclone has reduced the severe abrasion of the fan. Concrete, brick, or ceramic material linings in place of steel linings for stationary parts subject to abrasion are under experimentation.

(M) Sudden and large changes in sludge feed cause rapid changes in furnace temperatures with consequent long-time damage to refractory materials. Removal of the preheater is expected to reduce the operation and maintenance costs of incineration by about $0.33 per ton; fuel oil required for incineration has been reduced, only 1.5 gal now being necessary per ton of dry solids incinerated for heating up, cooling down, and holding temperatures in incinerators not in operation. Substantial fuel-oil savings were effected by installation of duct work so that available heat in operating furnaces could be utilized for heating up, cooling down, or maintaining temperatures in any desired unit. There is a possibility of the utilization of incineration heat for generating sufficient steam for all heating purposes and electric power requirements. Tests are under progress on incineration of filter cake to which has been added a portion of liquid sludge in the belief that, if the moisture content of sludge is reduced to 85% or 90%, dewatering on vacuum filters prior to incineration could be dispensed with economically, provided suitable incinerators are available.

(C) Extensive tests on sludge drying and incineration have led to the use of flash-drying equipment which satisfies the eight fundamental principles and factors for economical and efficient drying free from nuisances to prepare sludge for efficient combustion or for use as a fertilizer—maximum surface exposure, thorough agitation of the wet material, temperature potential of gas, velocity of drying gas, responsive and flexible temperature control, deodorization of gases vented from the drying operation, instantaneous water removal, preserva-

tion of fertilizer qualities. Heat-dried sludge should have a moisture content below 10% and be as free as possible from dust. The temperature of sludge at time of loading in cars should be below 100° F to prevent spontaneous combustion when stored in large quantities over a period of time. This is accomplished by screening, dedusting, and cooling in water-jacketed conveyers. It is necessary to raise the temperature of all gases and water vapor produced in the heat-drying of sludge to not less than 1,300° F in the presence of sufficient oxygen to effect complete combustion of offensive hydrocarbon compounds. Furnace spaces connecting flues and ducts of the sludge burner should be designed sufficiently large to accommodate excess gases such as nitrogen and oxygen. Tests with a commercially patented electrical precipitator indicate a removal of 97% of suspended solids from furnace stack gases. The presence of a large volume of visible water vapor in stack gases has not been objectionable in atmospheric pollution, if it is discharged at a height to permit complete dispersion. Where supplementary fuel is needed frequently in regular operation for drying and incinerating sludge in moderate size installations, such fuels in order of desirability, without reference to cost, are digester gas, natural gas, and fuel oil, or combinations thereof, and in large installations, powdered coal is very satisfactory and economical. At the Southwest Works, almost three years of operating plant tests on combining sludge drying with steam generation have proved its feasibility in providing cheap power, reducing furnace maintenance costs due to protection of refractories by water walls, and furnishing greater flexibility since furnace operation is not dependent on the drying load alone. More than half of the fuel is burned for steam generation so that sudden changes in drying load do not upset furnace conditions as they would if the furnace were designed for drying only. With all these improvements in drying and incinerating sewage sludge on record, still more developments are foreseen.

Operating Results.—

(B) For the first three years of plant operation, moisture in raw sludge was reduced from 96.54% to 92.77%; in digested sludge it increased from 87.8% to 90.89%; and in sludge cake it increased from 61.8% to 63.1%. Volatile matter in raw sludge, digested sludge, and sludge cake increased from 57.6% to 62.9%, from 41.5% to 47.5%, and from 38.8% to 42.9%, respectively. Heat content of sludge cake increased from 3,790 to 4,320 Btu per lb; filter rate in pounds of dry solids per square foot per hour decreased from 7.58 to 6.77.

(W) Elutriated sludge contained 44.4% volatile matter in solids; 3.8 volumes of Potomac River water to one volume of digested sludge gave a total alkalinity reduction equivalent to 7.5 volumes of wash water, which was due to countercurrent elutriation. Sludge cake contained 28.9% solids; filter rate averaged 5.9 lb of dry solids per square foot per hour.

(M) Moisture in raw sludge decreased from 92.85% to 91.89%, in the thickened sludge from 90.98% to 90.65%, and averaged 65% in filter cake. Volatile matter in filter cake increased from 52.1% to 58.8%. Filter rate was reduced from 5.50 to 3.90 lb of dry solids per square foot per hour. Filter rates of 5 to 10 lb per sq ft per hr were not found economical, since reductions in filter rates were attended by reductions in chemical requirements. Conditions

at Minneapolis-St. Paul indicated 2 lb per sq ft per hr as economical, based on total annual charges, including fixed charges, the basic influencing factors being type of sludge, cost of chemicals, etc.

Operating Costs.—

(B) The total cost of treatment excluding pumping (sedimentation, sludge disposal, and chlorination) was $6.92 per million gallons, or $19.38 per dry ton.

(W) Sludge elutriation, vacuum filtration, and sludge cake disposal costs only, excluding sedimentation and sludge digestion, were $1.33 per million gallons, or $7.69 per ton of dry solids in the cake.

(M) The cost of filtration alone decreased from $4.92 to $2.12 per dry ton and the cost of incineration decreased from $2.05 to $1.29 per dry ton. The total cost of collection, treatment, and sludge disposal decreased from $15.40 to $8.25 per dry ton; and the cost of sludge disposal varied between 40% and 45% of the total cost.

Use of Sewage Sludge as Fertilizer.—

(M) Tests are in progress on the use of sludge cake as a soil conditioner, the sludge being composted for a month or more to render it more friable and a better humus material for plant growth.

General.—Despite the numerous improvements in sewage sludge dewatering, incineration, and final use cited in the Symposium, it is of outstanding importance to note that each paper definitely indicates that still further developments are foreseen.

The Symposium shows that the many improvements were the result of combined efforts of sanitary engineers, mechanical engineers, chemists, and plant unit operators. It consists of papers on operation of sewage plants each serving populations of more than 500,000. According to the 16th Census of the United States in 1940 (29),[11b] there were only fourteen cities of 500,000 population or more. The latest comprehensive tabulation of sewage treatment plants in the United States (30) gives a total of 5,580 plants, of which 97% or 5,403 provide sludge digestion, and 3,522 or 63% have some form of sludge dewatering and disposal. Citation of these figures indicates how much more important and apropos the Symposium would have been if some papers had been included on plants serving populations of 20,000 to 200,000, which group would include a large percentage of the sewage treatment plants in the United States. For example, (M) data indicate that for sewage plants serving populations of 50,000 to 200,000 sludge digestion and drying on sand beds should be considered in comparison with dewatering by vacuum filters and incineration, based on type of treatment by which the sludge is produced, quantity of sludge to be disposed of, dependability of operation, and related factors.

The plant operation cost data presented in the Symposium leave much to be desired for better comparison and analysis. Cost of sewage treatment alone, exclusive of cost of sewage collection, should be furnished, on a million gallon basis and a dry ton sewage solids basis. In addition, cost of filtration should be given on a dry ton sewage solids basis, as well as cost of incineration, and other treatment unit operation costs as well, so that a definite dollars and

[11b] Numerals in parentheses, thus: (29), refer to corresponding items in the Bibliography at the end of the Symposium, and at the end of discussion in this issue.

cents value is presented; also a percentage of the total treatment cost should be made available for each unit.

As the (M) paper illustrates both in presentation of data and in stressing the point, studies can be made profitably, at all sewage plants, of economics of operation of present installation, possibilities of effecting economies by enlargement of present facilities at some future date, and economical rates for different types of sludge for the benefit of designers of future plants.

It is well to note that year after year at least one sewage treatment plant is presenting irrefutable data of the value of trained professional sewage treatment plant supervisors and operating personnel. The author of the (M) paper attributes satisfactory operating results to the relatively large number of professional engineers and chemists in supervising positions at his plant, necessitated by greater complexity of sludge filtration and incineration processes; to maintenance at high level at all times of interest and conscientiousness of entire personnel—particularly filter and incinerator operators; by control of filtration by Buechner funnel tests; hourly calculations of chemical dosages; shift competition; and utilization of new ideas. The splendid data presented in the (M) paper make one wonder whether other sewage treatment plants have done as well as they could, and should, in the employment of a number of qualified professional engineers and chemists as supervisors and in the training of plant operators.

Mr. Van Kleeck summarizes data on the conditioning and application of sewage sludge as fertilizer, a most important and promising phase of disposal of sewage sludge. Sufficient attention to this phase has not been given heretofore on an extensive cooperative basis. The progress made, especially since about 1928, indicates a successful solution with sound development.

Bibliography.—

(29) **Population—Number of Inhabitants—16th Census of United States,** U. S. Dept. of Commerce, Bureau of Census, Vol. 1, 1940, p. 32.

(30) **"A Summary of Census Data on Sewerage Systems in the United States,"** Public Health Repts., Vol. 57, March 20, 1942, No. 12, p. 420.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

DETERMINATION OF KUTTER'S *n* FOR SEWERS PARTLY FILLED

Discussion

BY C. E. RAMSER

C. E. RAMSER,[20] M. AM. SOC. C. E.[20a]—Since the paper appears to lack data on the effect of depth of flow on the value of *n* for tile pipes exceeding 12 in. in diameter, the writer offers the results of experiments on the flow of water in a 30-in. concrete tile pipe line under actual land drainage conditions in Murray County, Minnesota, during 1920.

For slope and current meter measurements, two manholes of 30-in. concrete tile were constructed 1,000 ft apart, the lower manhole being 234.5 ft above the outlet end of the tile line. Elevations of points at the manholes were established from which the elevation of the water surface could be determined by vertical measurements. From these elevations the slope of the water surface was computed, and current meter measurements were made with a rod-type current meter both in the lower manhole and in the ditch about 50 ft below the outlet of the tile. The manholes, consisting of 30-in. tile set up vertically, were built so as to cause practically no interference with the uniform flow of water in the tile. An 8-in. circular hole about in the center of the manhole was cut at a joint in the top of the tile. The manhole tiles were cemented around the bottom and at the joints to prevent leakage when the pipe was flowing under pressure.

The section of pipe where the experiments were conducted was straight and 1,000 ft in length. The tile pipe sections were laid with cemented butt joints along, and for some distance above, this section where a pressure head was expected.

Inspection at the two manholes indicated that the elevation of the abutting pipes at the joints varied from one half to three fourths of an inch. The tile was clean and free from any kind of growth.

NOTE.—This paper by C. Frank Johnson, Assoc. M. Am. Soc. C. E., was published in February, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1943, by Messrs. Thomas R. Camp, and Richard G. Coulter.

[20] Chf., Hydrologic Div., SCS, U. S. Dept. of Agriculture, Washington, D. C.

[20a] Received by the Secretary May 10, 1943.

Table 7 contains the results of the experiments. Measurements of the discharge and slope of the hydraulic gradient were made for depths of flow of from 0.67 ft to full with ratios of depth of flow to diameter of tile ranging from 0.27 to 1.00. The depth of flow was taken as the average of the depth at the

TABLE 7.—EXPERIMENTS ON THE FLOW OF WATER IN A 30-IN. CONCRETE TILE PIPE LINE

(D = Diameter of Tile; and A = Cross-sectional Area of Tile)

No.	Average depth of flow d (ft)	$\frac{d}{D}$	Average area of flow a (ft)	$\frac{a}{A}$	Average hydraulic radius R (ft)	Discharge Q (cu ft per sec)	Mean velocity V (ft per sec)	Slope S	Chézy coefficient C	Kutter coefficient n
					(*a*) UNDER PRESSURE					
1	2.50	1.00	4.91	1.00	0.625	29.96	6.10	0.00551	103.9	0.0133
					(*b*) PRESSURE AT LOWER MANHOLE					
2	2.48	0.99	4.90	0.99	0.638	23.00	4.69	0.00257	115.8	0.0122
3	2.48	0.99	4.90	0.99	0.638	21.21	4.33	0.00239	110.9	0.0126
4	2.45	0.98	4.87	0.99	0.666	20.16	4.14	0.00210	110.7	0.0126
					(*c*) FLOWING PARTLY FULL					
5	2.26	0.90	4.58	0.93	0.702	17.11	3.73	0.001345	121.3	0.01185
6	1.92	0.77	4.03	0.82	0.753	13.44	3.33	0.001126	115.0	0.0126
7	1.59	0.64	3.28	0.67	0.713	9.74	2.97	0.001017	110.3	0.0127
8	1.31	0.52	2.60	0.53	0.640	6.56	2.52	0.000966	101.3	0.0135
9	1.21	0.48	2.35	0.48	0.610	4.95	2.11	0.000966	86.9	0.0154
10	0.74	0.29	1.22	0.25	0.420	1.63	1.34	0.000976	66.1	0.0178
11	0.67	0.27	1.06	0.22	0.388	1.31	1.24	0.000966	60.1	0.0186

upper and lower manholes. The area of flow used in the computations was taken as the average of the areas at the upper and lower manholes, which explains the fact that the average depths of flow given in Table 7 do not correspond to the average area of flow. The hydraulic radius was taken as the average of the hydraulic radii at the two manholes.

Measurement No. 1, Table 7(*a*), was made when the pipe was flowing under pressure for the entire length of the slope course, and measurements in Table 7(*b*) were made when the pipe was flowing under pressure at the upper end of the slope course as determined from observations at the manholes at both ends of the course. Measurements in Table 7(*c*) were made when the tile was flowing partly full under no pressure, the average depth of flow ranging from 0.67 ft to 2.26 ft, and the ratio of $\frac{d}{D}$ ranging from 0.27 to 0.90. Attention is called to the rather consistent decrease in value of n in Kutter's formula for an increase in depth of flow. The values obtained for the two smallest depths of flow are slightly higher than the values given by the author for the somewhat smaller depths of flow for the 24-in. brick circular sewer D.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

APPLICATION OF SOIL MECHANICS IN DESIGNING BUILDING FOUNDATIONS

Discussion

BY MESSRS. D. P. KRYNINE, O. G. JULIAN, A. A. EREMIN, D. M. BURMISTER, ABRAHAM WOOLF, LAZARUS WHITE, GEORGE L. FREEMAN, AND TRENT R. DAMES, FREDERICK J. CONVERSE, STERLING S. GREEN, AND PAUL BAUMANN

D. P. KRYNINE,[37] M. AM. SOC. C. E.[37a]—Soil mechanics data concerning sands and clays are summarized in the first part of this interesting paper. These data refer to pressures and settlements and also to the methods of testing. It is regrettable that, in the second part of the paper, which deals exclusively with foundations on clay, this valuable information on sands did not find any application. The discussion hereafter follows the order of presentation of the material in the paper.

Angle of Internal Friction of Sand, ϕ.—The shear strength of sands (s) "increases approximately in direct proportion with the minor principal stress." This statement (see heading, "Soil Properties Responsible for Settlements: Stress-Deformation Characteristics of Cohesionless Soils"), is followed in the paper by the sentence: "The coefficient expressing this proportionality is called the angle of internal friction." Expressing this in symbols:

$$s = \phi \sigma_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The "angle of internal friction" is thus expressed in terms of pure numbers. Eq. 5 needs clarification.

Concentration Factor.—The authors are to be commended for stating very clearly that the principle of superposition is valid only if Hooke's law holds. The so-called "concentration factor" (or "concentration index"), as expressed

NOTE.—This paper by A. Casagrande, Assoc. M. Am. Soc. C. E., and R. E. Fadum, Jun. Am. Soc. C. E., was published in November, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1943, by Messrs. Jacob Feld, and Leonard Zeevaert; and June, 1943, by Messrs. Gregory P. Tschebotarioff, Guthlac Wilson, W. H. McAlpine, and Karl Terzaghi.

[37] Research Associate in Soil Mechanics, Dept. of Civ. Eng., Yale Univ., New Haven, Conn.

[37a] Received by the Secretary May 29, 1943.

in Eq. 1*b*, may be applied if the loaded area is very small, as in the highway subgrade. The writer admits that the integration of Eq. 1*b* over large finite areas is decidedly erroneous. A good substitute for the concentration factor is the rule proposed by A. E. Cummings,[38] M. Am. Soc. C. E. If there is a rigid underlying boundary at a depth, z, from the earth surface, the vertical pressure on that boundary may be computed according to the Boussinesq formula using $0.75\,z$ instead of z.

Varved Clays.—Fig. 6 represents a cross section of typical varved clay. It should be added that there are varieties of varved clays in which not two, but three, different materials are combined to form a clay mass. Geologists have studied the origin of varved clays,[39] particularly in New England. Engineering properties of varved clays in New England have been investigated by the U. S. Army Engineers in the Boston (Mass.) District and by the Connecticut State Highway Department, but nothing has been published to date (1943).

Time-Settlement Diagrams.—It is regrettable that Fig. 9 is not discussed by the authors. To the writer's knowledge, time-settlement diagrams, involving the fitting of the curves according to Professor Casagrande's method, have not been treated sufficiently well in engineering literature, and further clarification of Fig. 9 would be important. On the other hand, Figs. 8, 10, and 11 have been discussed adequately elsewhere.[33]

Example of a Settlement Analysis Based on Simple Soil Tests.—A simple method of determining the expected settlement without performing a consolidation test on a given sample is presented in the paper, providing only that results of tests on other clays are available. It is obvious, however, that, in addition to the tests enumerated in the paper, the specific gravity of grains must be determined or assumed.

Caissons.—To the knowledge of the writer, there has been no case of failure of deep-caisson foundations by shear (except, perhaps, during construction). Friction of a deep caisson against the earth mass considerably decreases the pressure on the base and increases the factor of safety. Uniform distribution of pressure at the base of a caisson as in Fig. 3(*c*) is rather a theoretical probability. In reality, because the camera is filled with concrete at the end of the work, probably there is a slight overloading at the edges of a caisson. An analogous situation is created when tunneling by the liner-plate method. In this case, there is an overloading under the legs of the frame and relief of pressure under the invert.

Differential Settlement Contours.—A uniformly thick clay layer decreases in thickness in proportion to the vertical pressure at a given point. The vertical pressure in the case of a nonrigid rectangular foundation is at a maximum at the middle of that rectangle. Hence, the maximum settlement is to be expected at the middle of a rectangular nonrigid building, especially if the central part of that building is overloaded as in Figs. 16 and 17. There is no way to avoid the settlement completely; but it is possible to decrease its absolute value by

[38] "Foundation Stresses in an Elastic Solid with a Rigid Underlying Boundary," by A. E. Cummings, *Civil Engineering*, November, 1941, p. 665.

[39] "Conditions of Formation of the Varved Glacial Clay," by E. Antevs, *Bulletin*, Geological Soc. of America, Vol. 36, 1925, pp. 171–172 (abstract); and other works of the same author and other geologists.

[33] "The Determination of the Pre-Consolidation Load and Its Practical Significance," by A. Casagrande, *Proceedings*, International Conference on Soil Mechanics, Vol. III, 1936, pp. 60–64.

making excavations of variable depths and to shift the excess of vertical pressure from one part of the building to another and thus to decrease the differential settlement. The quotation from Professor Casagrande's previous work[3] (under the heading, "Settlements of Buildings in Boston") will stand repeating. It is very true and should be kept in mind by the designers of foundations on clay deposits. In this connection, it is valuable to recall a statement, credited to Glennon Gilboy, M. Am. Soc. C. E., that driving piles into stiff clay is analogous to driving nails into a mirror to reinforce it.

The central contour lines of the differential settlement in Figs. 16 and 17 are elliptical; and the further a contour line is from the center of disturbance (overloaded central part), the less it is elongated and the more it approaches the shape of a circle. This is in accordance with Saint Venant's principle as applied in a horizontal plane.

The Liberty Mutual Insurance Company Building in Boston.—This building has a nonrigid foundation since the load is applied to the clay stratum by numerous caisson bells supporting piers, mats, and columns. Each bell is practically an independent load. For some reason point A, where the maximum settlement has been observed, is located somewhat eccentrically, closer to Stuart Street. This may be due: (*a*) To overloading or (*b*) to a change either in properties or in thickness, or both, of the underlying clay layer. It is very regrettable that in Fig. 17 observed contour lines only are shown, and no information with respect to the forecast settlement is given. This circumstance makes it impossible to judge how close the anticipated values of settlement were to the actual ones and to examine the causes of discrepancy, if any.

The New England Mutual Life Insurance Company Building.—This building differs from the Liberty Mutual Insurance Company in its weight and in the rigidity of the foundation, which, together with the structure itself, represents a rigid box. The building in question furnishes an excellent opportunity to show what a designer and builder of foundations must know about soil mechanics and how this knowledge should be used.

Prior to the final design of the building in question, the following was done:

(*a*) Extensive underground explorations were undertaken and the samples obtained in borings were tested in a laboratory (mostly for consolidation); and

(*b*) On the basis of this information a soil profile was prepared and analyzed. It was found that only two kinds of foundations are to be considered—a "floating" foundation or long piles reaching to hardpan. For economical reasons the former was preferred.

In general, a designer does not need to master the technique of borings and that of laboratory tests; but he must be able: (*a*) To order borings to be made at proper places and to see that they are taken properly; and (*b*) to see that the laboratory makes the proper tests and makes these tests properly. True knowledge of soil mechanics is needed in analyzing the soil profile. However, mere acquaintance with soil mechanics theories, not combined with extensive practice, and especially with what is called "sound judgment," is dead knowledge. In analyzing the soil profile, the designer must be prepared to answer

[3] "The Structure of Clay and Its Importance in Foundation Engineering," by A. Casagrande, *Journal*, Boston Soc. of Civ. Engrs., April, 1932.

the question: "What might happen to my structure if I build it this way?" Obviously, to answer such a question the Fourier series or the equation of the logarithmic spiral are not helpful. Similar details are required in studying soil mechanics for the first time, but, in applying this study to the design of structures, a thorough understanding of fundamental principles is required; and this can be seen very clearly from the examples of the second part of the paper.

A convincing proof of the necessity of combining theories with considerable practice is the ingenious detail proposed by Mr. Homer, as shown in Fig. 19(*b*). This detail simply could not have been worked out by a theorist without considerable building practice.

If soil mechanics were an exact science (which it is not), no measurement of settlements during and after construction would be necessary. In fact actual settlements would be exactly the same as the settlements exactly computed on the basis of exact soil and load information. In reality, settlements are carefully watched and measured at any project of importance. This is done to check computations made and theories underlying them and also in order to be able to take urgent measures during construction in the case of some abnormality. Particularly the authors are to be commended for the instructive nature of Fig. 22 which, to the knowledge of the writer, is the first in the literature to show both the elastic rebound due to the removal of an old building and the subsequent downward travel of the structure to reach, in the final analysis, the original ground surface.

In conclusion, the writer wishes to congratulate the authors for bringing to the attention of the profession the interesting and important results of their studies on foundations.

O. G. JULIAN,[40] M. AM. SOC. C. E.[40a]—A cogent and synoptic thesis on a subject which appears at long last to be emerging into the realm of rationality has been presented by the authors. The differences between soils and what are commonly known as "structural materials" are stated. However, since ultimately all structures are supported on soils (or rock), there appears to be no good reason for not considering the latter as structural materials. Also, it would appear evident that architects and structural engineers who are responsible for the safe and efficient design of buildings should be as well informed regarding the properties of soils as they are regarding other materials such as steel, concrete, brick, wood, etc., on which the stability of their structures depends.

As emphasized by the authors (see heading, "Introduction: Differences Between Soils and Structural Materials"), the statement that a given soil is "safe for" a certain unit load means very little. In addition to the maximum principal stresses, at least the orthogonal stresses and the relationship between stress, strain, moisture content, and time must be considered intelligently; otherwise, the structure may be subject not only to excessive settlement as a whole but to differential settlements which may possibly, in the course of time, wreck the superstructure. The authors correctly emphasize the point

[40] Chf. Structural Engr., Jackson & Moreland, Boston, Mass.

[40a] Received by the Secretary June 16, 1943.

that the supporting soil, foundation, and superstructure should be designed as an integral whole. In fact, these components ordinarily form a continuous system which is statically indeterminate to a high degree—the intelligent design of such a system requires knowledge of the properties of the soil and other materials, judgment, and experience to a greater degree than the ability to carry out precise mathematical analyses.

It is interesting to note from Fig. 2 the reversal from positive to negative of the change of slope of the strain-stress curve for sand at the point where the major principal stress exceeds the minor principal stress, that beyond this point the slope of the curve for "dense sand" is nearly constant up to the point where the major principal stress is twice the minor, and that dense sand increases in volume with increase in the major principal stress when the latter is three or more times the minor. This latter effect is due to the fact that in order to deform substantially the grains must slip over each other and in so doing cause expansion. The increase in volume indicates that the point of failure is approaching. As appears natural, the "elastic properties" and "shearing strength" of a given sand mass appear to be dependent almost entirely upon the ratio of the principal stresses—that is, on the degree of confinement. A loose liquid sand mass if confined may make excellent foundation material; however, emphasis should be placed on the importance of confining the entire mass. As stated by the authors (see heading, "Soil Properties Responsible for Settlements: Stress-Deformation Characteristics of Cohesionless Soils," conclusion (*m*)), serious settlements may result from lowering the ground-water table. Quantitative values pertaining to the curves given in Fig. 2 and definitions of "dense" and "loose" would be most instructive and interesting. It is hoped the authors will publish these data in their closing discussion.

With the exceptions of conclusions (*d*), (*g*), and (*i*), the thirteen conclusions pertaining to noncohesive soils (see heading, "Soil Properties Responsible for Settlements: Stress-Deformation Characteristics of Cohesionless Soils") appear to be intuitively evident, at least qualitatively.

Conclusion (*d*) is not evident; it would appear that at least in some cases the ultimate bearing capacity would increase as the first power rather than as the square of the depth. It would be interesting to know whether this conclusion is based on deductive reasoning or was arrived at empirically.

Conclusion (*g*) raises the question as to whether or not the theory of elasticity is valid in any degree when considering the stress or deformation condition of a mass of cohesionless soil. A persual of recent literature on the subject shows considerable divergence of opinion and indicates that the question is at least debatable. Since the theory of elasticity is based on continuity, Hooke's law, and its corollary, the principle of superposition, it would appear a priori that this theory cannot possibly be valid in the case under discussion. On the other hand, it is conceivable that, if subjected to considerable pressure, a confined mass of cohesionless particles may acquire, to some degree, continuity of action and elastic properties so that formulas based on the principle of superposition may be rough approximations and, in the absence of better methods, lead to conclusions that are of considerable value.

According to the theory of elasticity the deformation of an isotropic mass of semi-infinite extent, subjected to a uniformly distributed load over part of the free surface, depends on (1) the shape of the loaded area, (2) the area of the loaded area, (3) the intensity of pressure, (4) the elastic properties of the loaded material, and (5) the rigidity of the structure by means of which the load is applied.

For either a round or a rectangularly loaded area, provided the structure is flexible so that the intensity of loading is uniformly distributed, the average settlement, w_{aver}, may be obtained from Schleicher's formula:[41]

$$w_{aver} = m \sqrt{A}\, p \frac{1 - \nu^2}{E} \quad \ldots\ldots (6)$$

in which ν = Poisson's ratio; E = Young's modulus; and m = the "shape factor." The latter is a function of the ratio of length, a, to the breadth, b, of the loaded area. The value of m may be determined from the following approximate formulas:[41]

$$m = 0.37 + 0.61\, \epsilon^{-0.052\, a/b} \quad \ldots\ldots (7)$$

provided a/b is greater than unity and less than 100. In Eq. 7, ϵ = base of Napierian logarithms. Approximately, provided a/b is greater than unity and less than 10:

$$m = 0.98 - 0.027 \left(\frac{a}{b}\right) \quad \ldots\ldots (8)$$

For a circle $\frac{a}{b}$ may be taken as unity without substantial error. It may be noted that Eqs. 6, 7, and 8 confirm the authors' conclusion (*i*) as applicable to small areas. Also, as might be surmised from consideration of the resistance due to shearing rigidity, the settlement decreases as the ratio of the perimeter to the area increases.

If the structure is absolutely rigid and the loaded area is circular, Eq. 6 is applicable; but the value of the shape factor[42] becomes 0.89, making the settlement approximately 6% less than the average settlement of a flexible structure of the same shape and weight. From this it may be inferred that the rigidity of the structure has little influence on the average settlement.

Still assuming that the theory of elasticity applies, and further that the structure is absolutely rigid and that the bearing area is round (radius a), the intensity of pressure, q, immediately beneath the structure may be found from the well-known Boussinesq formula,

$$q = \frac{p}{2\sqrt{1 - \left(\frac{r}{a}\right)^2}} \quad \ldots\ldots (9)$$

According to Eq. 9, the intensity of pressure increases from a minimum of one half the average intensity at the center to a maximum at the perimeter; it

[41] "Theory of Elasticity," by S. Timoshenko, *Engineering Societies Monograph*, p. 338, Eq. 207; also references on p. 335.

[42] *Ibid.*, compare Eqs. 207 and 209.

approaches infinity as r approaches a. This latter intensity, of course, is not realized because the material yields and the pressure is redistributed. By Saint Venant's principle, this redistribution is accomplished without materially affecting the distribution at points some distance from the perimeter. This is well illustrated by Fig. 4(a). It may be noted that the distribution of pressure according to the theory of elasticity is the reverse of that which is intuitively evident for a footing on the surface of a sand mass and indicated by Figs. 3(a) and 3(b). However, comparison of these figures with Fig. 3(c) indicates that the slope of the distribution curve decreases at a rapid rate as $(b + h)\,\gamma$ increases and is practically zero for the case where $(b + h)\,\gamma$ equals p. It appears reasonable that further increase of $(b + h)$ would cause reversal of the slope of the distribution curve and that the intensities would approach the values, given by Eq. 9 and represented graphically by Fig. 4(a), as the breadth and depth increased indefinitely. Attempts by various investigators to express the variation in intensity, as the properties of the material and degree of confinement of the soil change, have led to the "concentration-factor" (α) concept. In addition to the Froehlick-Griffith formula (Eq. 1b),[43,44] the following empirical formula[45] proposed by D. P. Krynine, M. Am. Soc. C. E., for the case of an absolutely rigid round disk appears of special interest:

$$q = p\left(\frac{\alpha}{2} - 1\right)\left[1 - \left(\frac{r}{a}\right)^2\right]^{0.5\alpha - 2} \qquad \text{.....(10)}$$

in which q, r, and a have the same meaning as in the corresponding elastic Eq. 9 to which Eq. 10 reduces if α is taken as 3. If α is taken as 4, Eq. 10 states that the intensity of pressure is uniformly distributed, as illustrated by Fig. 3(c). As α assumes higher integral values, 5 and 6, the distribution curve becomes parabolic as in Fig. 3(b) and Fig. 3(a), respectively. When $\alpha = 8$, the distribution curve assumes a shape similar to that of a frequency curve. In addition to the aforementioned objections, it appears that the concentration factor, α, itself should be a variable increasing with r and decreasing with z. For example, in the case of a large foundation, the concentration factor for a given depth might be 3 near the center and about 8 as the perimeter is approached. Also, with r held constant, the concentration factor might be 8 near the surface and approach 3 as the depth increases indefinitely. This difficulty, however, might be overcome by a statistical approach—a soil mass of given properties and depth and loaded with a given intensity over a defined area being assigned an over-all constant concentration factor. Not only are careful settlement observations on a multitude of full-sized structures with known underground conditions required, but such observations should be supplemented by pressure-intensity measurements at various depths and horizontal locations. Since the distribution of pressure may change with time, these studies should be extended over a protracted period. F. E. Giesecke, M. Am. Soc. C. E., cites a case[46]

[43] "The Pressure Under Substructures," by J. H. Griffith, *Engineering and Contracting*, March, 1929, p. 113.

[44] "Distribution of Stresses Under a Foundation," by A. E. Cummings, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1072 *et seq.*

[45] "Pressures Beneath a Spread Foundation," by D. P. Krynine, *ibid.*, Vol. 103 (1938), p. 827 *et seq.*

[46] "The Distribution of Soil Pressure Beneath a Footing," by F. E. Giesecke and others, *Bulletin No. 43*, Texas Eng. Experiment Station, 1933.

in which, at the end of the construction period, values of σ_z under the corner and center of the rigid part of a footing bearing on clay were each approximately equal to 2,000 lb per sq ft; but, after the expiration of seven months, the pressure at the corner had increased to 2,240 lb per sq ft and that at the center had decreased to 1,760 lb per sq ft approximately. As stated by the proponents,[47] until sufficient field observations regarding the pressure distribution beneath foundations are made available, formulas such as Eqs. 1*b* and 10 must be considered rather hypothetical. Nevertheless, such formulas appear to be a step in the right direction and may give some idea of the distribution of pressure. Also, it would appear possible to derive corrective parameters which might be applied to elastic settlement formulas such as Eq. 6. However, as has been stated elsewhere by Professor Casagrande[48] in discussing clay masses, the problem is complicated by the introduction of layers of various materials in the underground. The introduction of comparatively stiff diaphragms or slabs, of course, will spread the load and materially reduce the settlement.

Sharp distinction should be made between such materials as sands, which are noncohesive, and clays, which, on account of the "molecular attraction" between the finely divided soil particles and the surface tension of the included moisture, are cohesive and therefore have intrinsic shearing strength even when not subjected to pressure. The clays possess definite elastic properties for loads of short duration; and, therefore, for such loads the distribution of stress and settlements may be calculated approximately from elastic formulas. However, since the major part of the settlement is due to the slow forcing out of the entrapped moisture over a period of many years and, since this process is proportional to the stress to which the soil is subjected, there is a tendency for the distribution of stress beneath a foundation bearing on clay to approach uniformity as time elapses. Either a rigid or a flexible structure supported on clay tends toward convexity downward. In the case of a rigid structure, this may produce stresses of considerable magnitude which should be considered in the design of the foundation and superstructure.

In addition to the elastic settlement which occurs upon application of the load, there may be additional settlement due to plastic and viscous flow.[49, 50] If the working pressure is not excessive, the plastic flow will be negligible; the viscous flow may continue at a diminishing rate indefinitely. Last, but by no means least, there will be settlement due to the entrapped water being forced out of the clay. This effect, due to the highly impermeable nature of the material, will continue at a diminishing rate for many years or even centuries. A method of determining the difference in void ratio corresponding to the difference in pressure caused by the removed overburden and the imposed load, the initial void ratio, and the ultimate magnitude and the rate of the settlement have been described clearly and in detail by the authors.

The foundation design for the New England Mutual Life Insurance Building appears to be almost ideal. Since the design is based on the most certain

[47] "Pressures Beneath a Spread Foundation," by D. P. Krynine, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 846, conclusion 12.

[48] *Ibid.*, Vol. 101 (1936), p. 1119 *et seq.*

[49] "Theory of Heat," by James Clerk Maxwell, Longmans Green, London, 1899, p. 295.

[50] *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 316.

principle in foundation engineering—that is, buoyancy—it should remove many uncertainties as to probable settlement and magnitude of stresses.

The basic requirement, stated by Professor Casagrande in 1932 and quoted in the paper (see heading, "Examples of the Application of Soil Mechanics to the Design of Building Foundations: General," paragraph (c)), is perfectly sound. To it might be added:

> "Design the foundation and superstructure so that they together may tip slightly but will not bend appreciably under the application of reasonable variations in the intensity of soil pressures."

These principles have been applied by the writer to the design of structures for the support and housing of units of machinery which are sensitive to differential settlement and which, if bent, would be rendered inoperative. In certain cases it may be found advisable to introduce steel trusses in foundation walls. Such foundations are comparable to boats inasmuch as they are, to some degree, buoyant. The New England Mutual Life Insurance Building is comparable to a boat floating on still water.

Without discussing the question of pile foundations in detail, it may not be out of place to mention that the action of a group of friction piles, or piles bearing on an intermediate hard layer under which the soil may yield, is subject to the principles stated in the paper. The soil is subjected to a distributed load of varying and somewhat speculative intensity comparable to that which would be introduced by a foundation mat. The bearing capacity of such a group of piles cannot be evaluated rationally as the capacity of a single pile times the number of piles. Also, since their driving disturbs the clay, a group of friction piles in clay may aggravate rather than minimize settlement.

The authors, in this paper, have made a definite contribution to foundation and structural engineering. It is believed that the value of the paper would be enhanced considerably if, in their closing discussion, they would include a tabulation of range in, and typical values for, void ratios, shearing strength, compressive strength on unconfined specimens, Young's modulus, and Poisson's ratio for clay and other soils.

A. A. EREMIN,[51] ASSOC. M. AM. SOC. C. E.[51a]—In this paper the authors have opened an interesting discussion of soil mechanics as applied to foundation design. It is true that correct application of soil mechanics requires experience and judgment. Therefore, a discussion of cases encountered in practice is highly desirable.

Two groups of problems in soil mechanics are generally involved. The first is the distribution of stresses produced by the superimposed loading, and the second is the computation of the resulting deformations in soil, or the settlement of foundations.

Various stress distributions in soils have been classified, the most important being those based on the Boussinesq formula cited in Eq. 1*a*. D. P. Krynine,[52]

[51] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[51a] Received by the Secretary June 21, 1943.

[52] "Pressures Beneath a Spread Foundation," by D. P. Krynine, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 827.

M. Am. Soc. C. E., has made a valuable contribution by simplifying the graphical application of the Boussinesq formula, and F. Kögler and A. Sheidig [53] have developed interesting simple methods of computing stresses in soil, the final results of which are in close agreement with that obtained with the Boussinesq theory.

In computing the critical failure stresses in soil, shown in Figs. 13(*a*) and 14(*a*), the authors have assumed that the shear stresses are determined by the horizontal tangent to the circle of stresses. A more exact expression of shear stresses in a homogeneous material is determined from the curved line, or from the sloping tangent shown by S. Timoshenko.[54]

For several reasons, the computation of deformations and settlement yields indefinite results in soil mechanics. The computed deformations can be verified only by measuring the settlements of completed structures, which measurements generally are not readily obtainable. Furthermore, the settlement of a foundation varies not only with the physical properties of the soil, but also with the methods of constructing the foundation. For instance, it is well known that the effect of upheaval in a deep excavation may be reduced by driving the cofferdam sheathing to a greater depth than that generally required for the lateral stability of the cofferdam. Likewise, the effect of upheaval may be reduced by digging the last layer immediately before building the foundation; or it may be reduced by pre-stressing the soil with a temporary loading.

Computation of settlement by plotting graphs of the ratio of voids in soil, as shown in Fig. 12, may produce results that vary within a wide range. In applying this method, the authors did not take into consideration the facts that the stresses in soil vary with a variation in depth of foundation, and also that at various horizontal planes under a foundation the stresses are not uniform.

The authors' conclusion that the settlement of a foundation can be reduced to a tolerable magnitude by removing a quantity of soil which, in weight, is approximately equal to that of the superimposed structure should be considered true only in the case of a limited depth of foundation.

It is true, as the authors state, that the settlement of building foundations is greater in the middle of the structure than that at the walls of the structure. However, the reverse is quite possible. For illustration, reference may be made to experience with the government building in Cairo, Egypt, where the foundations settled less in the middle than at the walls. The Cairo building was designed in 1930, careful consideration being given to the principle of soil mechanics.[55]

D. M. BURMISTER,[56] ASSOC. M. AM. SOC. C. E.[56a]—The achievement possible in designing and constructing satisfactory, as well as economical, building foundations by the proper application of the principles of soil me-

[53] *Die Bautechnik*, 1929, p. 268.
[54] "Strength of Materials," by S. Timoshenko, D. Van Nostrand, New York, N. Y., Vol. 2, 1941, p. 712.
[55] *Die Bautechnik*, 1935, p. 93.
[56] Asst. Prof., Civ. Eng., Columbia Univ., New York, N. Y.
[56a] Received by the Secretary July 26, 1943.

chanics is demonstrated in this paper. The practical value of reliable methods of reducing the total magnitude, and particularly differential settlement, is obvious. No less interesting and important are the actual design features and methods of construction for the two structures analyzed in the paper.

In such situations, where the methods described are applicable, the uncertainties involved in settlement investigations may become somewhat less important, and the designer can proceed with greater assurance of satisfactory results on the basis of the available information on subsurface conditions, such as usually can be obtained economically by present methods of soil explorations. For other situations, where these methods are not directly applicable, the uncertainties involved in the settlement investigation may become a major concern in design. It was emphasized by the authors that subsurface conditions cannot be learned in sufficient detail, even by the most elaborate explorations, to serve as an entirely adequate basis for the selection, design, and installation of a satisfactory and economical type of foundation. Consequently, as stated, the results of a settlement investigation should be expressed by a range of values, which include estimates of the possible limits of all factors involved.

The writer considers the following as being the more important sources of uncertainty and error in settlement investigations:

(1) The possible variation in thickness of the compressible stratum between borings from an assumed stratification;
(2) The natural state of stress in the clay deposit or overburden pressure, p_n;
(3) The consolidation properties of the clay;
(4) The foundation stresses in the clay stratum, p_f; and
(5) The settlement characteristics of stratified soil deposits.

Information on the first three items is obtained from subsurface explorations and laboratory soil tests. The sources of uncertainty are due primarily to the necessity of selecting average values on the basis of limited information, and to unknown variations in subsurface conditions between borings which must be assumed, guided by experience in the given area and by judgment. Fairly reasonable values usually can be assigned without much difficulty to the possible limits of variation of these quantities. Their influence, expressed as a range of settlement values in terms of percentage variations from the average, can be estimated by taking partial derivatives of the basic settlement equations with respect to each variable quantity.

In addition to the notation of the paper, let: S = settlement; H = thickness of clay layer; C_c = compression index; C_v = coefficient of consolidation; e_n = void ratio of material in its natural state; p_n = overburden pressure; p_f = foundation stress in the clay stratum; U = consolidation, expressed as a percentage of the total; t = time; T_v = time factor corresponding to values of U; and Δ = variation in any quantity.

For total settlement:

$$S = H\left(\frac{C_c}{1+e_n}\right)\log_e\left(\frac{p_n+p_f}{p_n}\right) \quad \ldots\ldots\ldots\ldots\ldots (11)$$

and, for time rate of settlement:

$$\text{Settlement (in percentages)} = S\,U \quad \ldots\ldots\ldots\ldots\ldots (12a)$$

and

$$t = \frac{T_v\,H^2}{C_v\,(1 + e_n)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12b)$$

The influence of variations of quantities on settlement is treated as follows:

(1) For percentage variations in thickness ($\pm\,\Delta H$)—

$$\text{Settlement} = \frac{\partial S/\partial H}{S}\,100\%, \quad \text{or} \quad \pm\,\frac{\Delta H}{H}\,100\% \quad \ldots\ldots\ldots (13a)$$

and

$$\text{Time} = \frac{\partial t/\partial H}{t}\,100\%, \quad \text{or} \quad \pm\,\frac{2\,\Delta H}{H}\,100\% \quad \ldots\ldots\ldots (13b)$$

(2) For percentage variations in overburden pressure ($\pm\,p_n$, due to possible variations in unit weights or in ground-water elevations)—

$$\text{Settlement} = \pm\,\frac{\dfrac{p_f\,\Delta p_n}{p_n\,(p_n + p_f)}}{\log_e \dfrac{p_n + p_f}{p_n}}\,100\% \quad \ldots\ldots\ldots\ldots\ldots (14)$$

(3) For percentage variations in consolidation properties $\left(\pm\,\Delta\,\dfrac{C_c}{1 + e_n}\right)$—

Settlement, variation in reduced compression index

$$= \pm\,\frac{\Delta\left(\dfrac{C_c}{1 + e_n}\right)}{\dfrac{C_c}{1 + e_n}}\,100\% \quad \ldots\ldots\ldots\ldots\ldots\ldots (15a)$$

and

Time, variation in the coefficient of consolidation

$$(\pm\,\Delta C_v) = \mp\,\frac{\Delta C_v}{C_v}\,100\% \quad \ldots\ldots\ldots\ldots\ldots\ldots (15b)$$

These factors actually may combine to produce favorable or adverse combinations, and the actual settlements may be decreased or increased accordingly with respect to the predicted values. Although statistical methods do not help much in arriving at a reasonable probable combination, it is suggested that the square root of the sum of the squares of the percentage variations expressed by Eqs. 11 to 15 will give a reasonable probable range of values, due to a combination of these factors, which may amount to $\pm$ 25% to $\pm$ 50%, depending on conditions. Variations in conditions at any point influence conditions at all neighboring points because the structure and the supporting soil behave as a continuum. Experience has indicated that the average soil properties in many stratified deposits do not vary as much as one might expect in a horizontal direction within each stratum, although they

may vary in a vertical direction in a more or less consistent fashion within the given stratum, due to changing conditions during deposition.

The sources of uncertainty and error under items (4) and (5), which are inherent in every settlement analysis, may be of much greater consequence, because they affect primarily the estimates of differential settlement and the rate of settlement in the early stages, which are needed for appraising the situation as to what should and can be done. These uncertainties arise from the fact that the foundation stresses in natural soil deposits and the settlement characteristics may be quite different from those assumed in the simplified theoretical approach used in the solution of settlement problems. It is difficult and even impossible at present to assign numerical values to the influences of these uncertainties.

The writer is in agreement with the basic soil mechanics principles underlying the design of foundations presented by the authors, but wishes to add a few remarks regarding some of them. Under item (4), the stratification of a deposit, the degree of rigidity of the structure, and the depth of the foundation below the surface of the ground have important influences on the intensity and distribution of stresses in a compressible clay layer. It must be realized that at best the Boussinesq stress equation, as stated by the authors in item (*g*) (see heading, "Introduction: Stress-Deformation Characteristics of Cohesionless Soils"), can give only a rough approximation for the stress conditions and that the degree to which the law of superposition of stress and strain is valid in a sand stratum overlying clay is open to question. Nevertheless, both represent most important tools in settlement investigations; otherwise, a solution would be impossible. Therefore an assumption of their validity must be made, even though accepted with reservations. It is necessary to distinguish between near surface phenomena in connection with relatively small bearing areas on sand, where the theory of elasticity does not apply, and the intensity and distribution of stresses transmitted through a sand stratum into a clay layer, where the agreement may be much better, provided the solution for a two-layer soil system were known.

Under item (5), the settlement characteristics of stratified soil deposits, both as to total magnitude and rate of settlement, are affected markedly by conditions, because the structure and the supporting soil behave as a continuum. They will tend to adjust themselves in accordance with a kind of "principle of least work," so that the weaker spots tend to relieve themselves of stress and transfer stress to adjacent stronger areas with corresponding adjustments in settlement. The degree of rigidity in the structure tends to reduce differential settlement materially, because the foundations of the actual structure and the supporting soil cannot "follow down" exactly in accordance with the theoretical estimates based on the assumption of individual foundation loads, which are free to move vertically without restraint. The question is: What is a satisfactory measure of the rigidity of a structure, and how much reduction in differential settlement can be achieved economically?

The authors stated in item (*k*) that sand layers provide an excellent medium for spreading load and for reducing differential settlements. The restraints and resistance to deformation of a fairly thick sand stratum overlying

a compressible clay stratum also may reduce the total settlement very materially. It is believed that the solution to this problem is best approached empirically from the settlement rather than the stress "angle" by comparing actual settlements with predicted settlements, after making proper allowances for the other factors discussed herein. When a number of structures have been observed for which the subsurface conditions are fairly well known, an empirical coefficient or multiplying factor may be determined by plotting the ratio of the actual to the predicted settlements against the ratio of the width of the structure to the thickness of the reinforcing sand layer. The decrease in settlement with depth below the adjacent ground surface, as stated in item (*j*), may be approached in a similar manner.

In addition to the foregoing factors, which tend to decrease settlement materially, two other factors must be mentioned. Two-dimensional consolidation (see Biot's solution[57]) in contrast to the simplified approach in the one-dimensional theory and the lateral subsurface displacements (lateral bulging effects), which always accompany and contribute to the total magnitude of settlement, may tend to increase settlement somewhat at the walls and decrease differential settlements in the early stages of consolidation.

In order that engineers in other localities may compare their situations with those reported, the authors should include in their closing discussion information concerning the physical properties of the soils in the deposit. It would be of immense value to the engineering profession if the authors would include what they have judged to be the possible variations and the limits for the different factors involved in the settlement investigations and what they have learned from the actual settlement observations of the two structures as a full-scale check on their assumptions as to foundation stresses and settlement characteristics of the deposit.

In conclusion the writer wishes to congratulate the authors on their valuable contribution to applied soil mechanics.

ABRAHAM WOOLF,[58] ASSOC. M. AM. SOC. C. E.[58a]—Together with the architects and owners, the authors should be complimented, not only for their excellent study and report, but for their courage in the design and actual construction of a type of foundation radically different from the usual Boston type of foundation. From the settlement results shown in Fig. 15, it is evident that the floating type of deep foundation is the solution to the problem as applied to the conditions in the City of Boston. However, the conclusion that this type of foundation is the general solution in every locality, where at least similar conditions exist, is not justified reasonably. The authors were in a position to assume responsibility for the floating type of design, with the certainty of the resulting settlements as shown in Fig. 15, only after extensive study and observation of the action of several similar structures in Boston. It is to their credit that they correlated their observations and experiences in the field, together with laboratory practice, and prepared a definite procedure

[57] "Consolidation Settlement Under a Rectangular Load Distribution," by M. A. Biot, *Journal of Applied Physics*, Vol. 12, No. 5, May, 1941, pp. 426–430.

[58] Structural Engr., Albert Kahn, Inc., Detroit, Mich.

[58a] Received by the Secretary July 30, 1943.

for the design of a floating type of foundation. The experience of the authors was the prerequisite to the design. This fact alone should call attention to how important it is for municipalities, universities, and engineers to keep accurate records of the settlements of important structures and also to make laboratory soil tests so that any design which departs from the usual type would be justified in a particular locality.

In underconsolidated clays, such as exist in some parts of Detroit, Mich., there is a possibility that the installation of a floating type of foundation might cause settlements beyond those expected. The character of clays, preconsolidated under loads less than the overburden, presents a problem of sensitive nature, and the floating type of foundation needs careful study if adapted to such clays. The installation of a deep basement, no doubt, reduces the load on the substrata below and at the same time provides sufficient depth to allow for the design of a rigid foundation; but the installation disturbs the firm upper strata and also provides additional drainage immediately below the concrete foundation. The improved drainage increases the rate and amount of consolidation by relieving the excess hydrostatic pressure in the underconsolidated clay. The result, of course, would be increased settlements. However, in normal clays, which are preconsolidated under heavier loads than the existing overburden, no such excess hydrostatic pressures exist, as in the underconsolidated clays, and, even though the deep basement does provide a better drainage under the foundation, there can be no evidence of any additional excessive settlements because the stress condition in the substrata is in balance due to the fact that the excavated loads are approximately equal to the building loads.

Conclusion (5) reads,

> "Differential settlements are best controlled by designing the foundation and the superstructure as an integral unit, with special attention to stiffness. Where this is not acceptable, design for stiffness must be confined to the foundation, which increases the cost and decreases its effectiveness."

This statement becomes of interest to the designing structural engineer because it is his design which controls the internal differential settlements. The writer wishes to classify the differential settlements into two types—the external and internal. External differential settlements are measured at relative points on the periphery of the building, the internal are measured at points within the structure. The purpose of segregating the differential settlement into two types is to help analyze the condition of equilibrium of the foundation, itself, as a structure. The variation in stiffness of a rigid substructure has no material effect on the external differential settlements; but it definitely controls the internal condition. Fig. 23 shows a maximum external differential settlement of 1.16 in. between the two long sides of the building on Boylston Street and Newberry Street, whereas the maximum internal differential settlement on the walls, running crosswise, does not amount to more than 0.5 in. It may be that the Boylston Street subway acts to reduce the settlement rate on the Boylston Street side, thereby increasing the differential on the two long sides. The almost balanced condition of the two shorter sides indicates equilibrium.

The internal differential settlements and resulting distortions are a measure of the shears and moments of component parts of the rigid foundation. In Fig. 23 three of the cross-walls show tension on the top, and the fourth shows tension on the bottom. The writer has spot-checked the wall showing tension on the bottom and found that the moment developed is approximately 635,000,000 in-lb based on a differential deflection of 0.2 in., requiring about 60 sq in. of reinforcing steel at 24,000 lb per sq in. in the bottom of the wall. Since the actual amount of steel installed was not known, it was felt that it would be an interesting check on the original design.

At present, there appears to be no definite answer to the question, "How stiff to make the foundations."[59] A rational approach to the problem would be to design the integral members of the foundation so that structural stability is insured and then to determine what elastic distortions these members will develop under this stability. These distortions can be checked with the soils engineer, and, of course, it would be his responsibility to advise whether the structure is too stiff or too flexible or whether the original assumption of soil reaction distribution needs revision. Also he must decide what adjustment should be made to satisfy the elastic distortion in the soils. The coordination of the instantaneous elastic yield of steel and concrete with the time lag in the elastic yield in clays is perhaps somewhat beyond the present-day developments. Attempts have been made to assume upper and lower limits in the soil and then to interpolate the results obtained by means of reasoning at least to set the direction of the stress.[60] The theory involves the so-called K-values (moduli of subgrade reaction which are given in pounds per square inch per inch), but these values do not introduce the time effect, which is quite important in the analysis of both the structural stability of the foundations and the soil substrata beneath.

In an investigation of recent date regarding the structural stability of a building which had undergone considerable total and differential settlements and still appeared to be sound and not under any serious condition of distress, the following was deduced:

(1) In a distorted, rigid, steel structure, in which the distortions are due to the displacements of column reactions because of settlement, if the calculated stresses in the structure, due to this distortion are at, or beyond, the yield point of the steel, the structure, itself, will undergo elastic relief and these calculated stresses will not exist. The actual stresses in the structure are reduced, by this relief, to the yield-point values or below; otherwise, the entire structure would be in a state of complete collapse, as indicated by the high calculated stresses.[61]

(2) In a concrete structure a similar condition prevails, but the relief is developed by rupture of the concrete in tension, made evident by the appearance of cracks at those points where the reinforcing steel is omitted. The

[59] "Economics of Rigid Frames for Building Foundations," by K. R. Kimball, *Engineering News*, September 26, 1935, p. 427.

[60] "Stresses in Concrete Runways of Airports," by H. M. Westergaard, presented to Highway Research Board, Washington, D. C., December, 1939; published by Portland Cement Assn.

[61] "Theory of Limit Design," by J. A. Van den Broek, *Transactions*, Am. Soc. C. E., Vol. 105 (1940), p. 638.

elastic range between the yield and rupture points for concrete in tension is so close that there is no opportunity for relief to develop without rupture.

These observations are mentioned merely to indicate that, in the design of a rigid concrete substructure, the possibility of a tension stress in the concrete should be eliminated and all concrete members subjected to such possible conditions of stress should be reinforced both top and bottom. The observations also indicate that moderate settlements are not too serious when applied to an analysis of the stability of a structure.

Conclusion (*m*) does not imply that lowering the ground water decreases the bearing capacity of the loose sand. Rather, it refers to the disturbances in the sand structure due to the lowering of the ground water. The writer has made observations where the raising of the ground water has decreased the ultimate bearing capacity of dense sands.[62] The observations indicate ratios for different kinds of sands of $\frac{\text{bearing capacity, dry}}{\text{bearing capacity, wet}}$ as follows:

Item	Grade of sand	Ratio
(*a*)	Nonuniform, well graded	2.83
(*b*)	Uniform, medium grain size	1.98
(*c*)	Uniform, fine grain size	1.52

In Fig. 15(*b*), curve IV is established in its position relative to curves III and V without mentioning the fact that the total settlement of the particular point was known six years after the building was finished and the extrapolation was established for curve IV with those data. In several settlement studies settlement readings were not taken from the construction completion date to the point where the differential distortions become troublesome; and it was impossible to establish the total settlement definitely, particularly where there have been no permanent bench marks and the site immediately surrounding the structure had undergone subsidence. In this connection, it would be of value for any building to establish at least one level of the completed structure definitely tied in with some permanent city bench mark, instead of with a curb, manhole, or any other flexible point. The authors might clarify this point with regard to curve IV.

In discussing Fig. 18(*a*), the authors remark that,

> "It was specified that, if it were discovered, during the progress of construction, that the hard clay in some areas approached the sub-basement slabs, it would be necessary to excavate to a sufficient depth and replace the hard clay by soft organic silt-clay."

The radical deviation from the usual construction practice makes this clause difficult to accept, particularly with respect to the method of backfilling a soft organic silt-clay.

Fig. 19 shows the unsymmetrical footings of the outside walls of the New England Mutual Life Insurance Company Building. The final design indicates

[62] "An Experimental Investigation of the Relation Between the Bearing Capacity of Dry and Wet Sands," by J. P. Luby and A. Woolf, thesis presented to Massachusetts Institute of Technology, Cambridge, in 1928, in partial fulfilment of the requirements for the degree of Bachelor of Science.

a procedure better suited to deep foundations which require a minimum disturbance than to areas outside the building or structure. This type of wall footing is usually standard practice in underpass and opencut tunnel design. The method of thickening the slab under the wall without indicating structural continuity at the joint between the slab and the wall is not clear. It would be interesting to know the distribution of the soil reaction under this type of footing in order to establish design procedure for the structural stability of the slab and wall.

LAZARUS WHITE,[63] M. AM. SOC. C. E.[63a]—A large part of this paper is devoted to theoretical soil mechanics, particularly as applying to unconsolidated clays. Considerable valuable information regarding the nature of building settlements and actual records of settlements of buildings in the Boston district are given.

The writer does not wish to discuss theoretical soil mechanics, although he regrets that the use of photoelastic methods as applied to soil mechanics is not mentioned. Also, he regrets that the plottings of the lines of equal vertical pressure beneath the building foundations, described in detail, are not given. For some reason, soil mechanics, as exemplified by the paper, seems to have relegated Boussinesq to the rear and eschewed the graphical representations of pressures so prominent in soil mechanics of only a few years ago. In fact, by a circuitous route, the authors arrive at a conclusion so simple that one wonders whether all the elaborate mathematics of present-day soil mechanics is necessary. All one needs to do, according to the authors, is to excavate from the cellar and subcellar of the proposed buildings a weight of material equal to the weight of the proposed building. The structure can then be erected on mats of reinforced concrete to gain a foundation free of undue settlements, with additional cellar and basement space, all for the cost of a few thousand yards of additional excavation—surely a small price to pay for the benefits realized.

However, in justice to the authors, the writer wishes to call attention to their statement (see first paragraph of "Introduction"),

> "It cannot be emphasized too strongly that the correct application of the results of such an investigation to the design of earthworks or foundations will, in addition to a thorough knowledge of soil mechanics, always require experience and judgment."

This conforms to an old test of any engineering design. Does it conform to good practice?

The writer hesitates to take issue with the authors as to the theories of soil mechanics and confesses that he has been unable to follow the elaborate mathematics of this paper. Nevertheless, there are some basic principles by which to test the conclusions; also, there is considerable experience as to what is good practice. The writer begs to differ with the conclusions of the paper, particularly Conclusions (1), (2), and (6).

[63] Pres., Spencer, White & Prentis, Inc., New York, N. Y.

[63a] Received by the Secretary July 31, 1943.

There is an old recognized principle that, in a structure, the loads tend to go to the hard points. As applied to a building with numerous columns extending to cellar grade, the loads can best be picked up by the foundations directly beneath the column bases. The best foundation is based upon this principle. For instance, in the first Chicago (Ill.) "skyscrapers," mat foundations were used on the Chicago clay with unsatisfactory results; then the Chicago caissons were introduced, which carried the loads directly from the columns to hardpan or rock. Later, various types of steel piling and caissons were used successfully. Conclusions (1) and (2) would set the profession back forty years.

When an excavation is made, various disturbances take place. The groundwater level usually is lowered in an irregular way, and, as the authors state, the interior bulges, which may or may not be serious. The placing of sheeting around the exterior of the excavation creates disturbances.

Usually it takes several weeks before the foundation can be completed, during which time the interior is exposed to the weather. It dries out if exposed to the sun, or is saturated if exposed to the rain.

Now if, instead of designing a foundation to carry the loads directly to a secure stratum and by-passing the weak upper stratum, the engineer resorts to a "raft" or "mat" foundation and attempts to avoid differential settlements by an elaborate system of reinforced concrete girders, what risks are run. In the first place, uniformity of pressure is impossible. As amply demonstrated by photoelastic experiments and the application of the Boussinesq formulas, differential pressures bring about differential settlements which may be very damaging. This is covered quite fully in the Progress Report of the Special Committee on Earths and Foundations.[64]

It happens that the Boston clay has been studied intensely over quite a long period, and its qualities are well known; it is geologically a simple formation founded upon hardpan or impervious rock. Calculations of settlements of buildings on Boston clay, computed by the use of the Boussinesq equations and the consolidation test and methods devised by Professor Terzaghi, have agreed fairly well with observed settlements, but nevertheless in other and more complex areas the engineer should be on his guard; consolidation is a drainage problem and the condition must be simple and capable of analysis. However, there are wet areas with difficult conditions, such as that adjacent to the Great Lakes where an oxidized yellow clay prevents upward drainage and an underlying limestone may keep the intermediate soft blue glacial clay under an artesian pressure built up to high values—in some cases by natural gas.

Deposits of clay are often associated with sand pockets under pressure. Such pockets may complicate the drainage of the clay so as to defy computation. Again, an underlying bed of porous limestone may keep the clay under such hydrostatic pressure as to prevent consolidation.

At one location in the Hudson Valley a heavy glacial clay deposit of varved clay is still very soft after a period dating back to the glacial age. This is because the wet blue clay is confined between two layers—an upper, oxidized, very tight yellow clay, and a lower porous limestone under pressure from ground

[64] *Proceedings*, Am. Soc. C. E., May, 1933, p. 777.

water in a near-by mountain. In the vicinity of Detroit, Mich., a similar condition prevails—an upper crust of yellow clay is underlain by a thick deposit of soft blue clay, also under pressure from a porous layer above the rock floor. For these conditions, the writer knows of no formula which could give the correct rate of consolidation for the blue clay. In both cases, structures placed on the upper yellow clay came to grief—in one case, the blue clay beneath a heavy building squeezed into an adjacent excavation, and in the other case a structure on a heavy mat, in turn resting upon an oxidized layer, settled and upset because of plastic flow in the underlying soft blue clay. The subsoil in any city is subject to changes; it is usually being drained, with lowering of ground water, or a broken water main may cause a rise in the ground-water level.

To the writer, it seems wise to design any important structure with its foundations extended to rock or some deep, sound stratum. There are several methods available. The cost may be a little higher than for a spread footing or mat foundation, but it may prove to be cheap insurance.

The writer has every respect for the authors but does not believe that soil mechanics has advanced far enough to furnish the basis for the design of mat foundations upon unconsolidated strata such as wet clays or silts. Engineers cannot predict confidently what settlements will occur. Such mat foundations are subject to many dangers—from neighboring structures, from neighboring excavations and pile-driving operations, and from the shifting of floor loads.

Methods of installing foundations, particularly to great depths, have been so improved that in most instances the old-fashioned shallow foundations have been superseded. However, in some districts there are thick strata of unconsolidated clays and silts, and the risk of uncertain and irregular settlements of shallow footings must be taken. Soil mechanics may be invoked in such cases to diminish the risk.

Soil mechanics alone is not sufficient for choosing a site or designing a foundation. It is a great aid to the engineer, but not his master. For instance, a building may be "plumped" in the middle of an extinct pond in which, at depths affected by the structure, are layers of unconsolidated peat. Such localities may be detected by the practiced eye of a trained geologist.

The shape of a building is a great consideration in the design of a foundation —whether it is of uniform height or of a tower design, and whether or not it contains courts and projecting wings. Such dangers may be revealed by a plotting of the iso-pressure lines computed from the Boussinesq formula.

GEORGE L. FREEMAN,[65] M. AM. SOC. C. E.[65a]—Foundation engineers will find this paper particularly valuable because it follows through, from a statement of the basic principles of soil mechanics affecting the designs, to a description of the "finished products," with records of the settlement phenomena accompanying and following the construction periods. The description of the foundation of the New England Mutual Life Insurance Company Building is of interest, with its record of subgrade heaving and surrounding soil movement.

[65] Cons. Engr. (Moran, Proctor, Freeman & Mueser), New York, N. Y.

[65a] Received by the Secretary July 31, 1943.

and the adaptation of the construction methods to job conditions. It is obvious that close control of construction methods is essential on such work; otherwise, subsoils and bearing strata may be disturbed and much of the care and skill expended in the design may be nullified.

It may be assumed that the two substructures created by the authors reflect to full extent the benefits of thorough soil studies and their application in the designs. From the performance records, it is evident that both structures have proved satisfactory in service, and the designers are to be congratulated on their success.

The settlements of the belled piers under the Liberty Mutual Insurance Company Building are partly dependent on the thickness and continuity of the hard crust which overlies the deep, soft clay bed. It would be interesting to know what steps were taken to prove the integrity of the crust under each bearing area. The value of the 10-ft, hard bearing stratum in distributing load concentrations to the underlying soft clay is evidenced by the regularity and symmetry of the settlement contours in Fig. 17. In both this design and that of the foundation of the New England Mutual Life Insurance Company Building, the designers have taken full advantage of the fortunate existence of the hard crust 40 ft below the ground surface, without which both designs would have been materially different from those shown.

The writer prefers the type of design used for the foundations of the New England Mutual Life Insurance Company Building, as its wall arrangement, if strongly reinforced, can lend great stiffness to the substructure, assisting to smooth out local deflections. Had there been no bearing crust at the 40-ft depth, a full mat design would have been indicated, with the walls modified to some extent to provide proper stiffness.

The provision for future increases in heights of certain sections of the New England Mutual Life Insurance Company Building greatly complicated the problem of designing a floating foundation. With soil excavated, as stated, in an amount to equal the weight of the building, including its future extensions, it would appear that a greater differential of settlement than shown might be expected between the present lightly and heavily loaded areas, unless some compensating factors were introduced into the design. Footing widths shown in Fig. 18(*a*) were apparently designed for ultimate building heights. Would the authors please clarify this point, and indicate the locations of the building sections of varying heights mentioned in the paper?

The tabulation of vertical movements of the hard clay stratum shows that point 5 was up 2.9 in. on October 12, and that other points continued to rise until October 30. It seems from the record that the upward movement must have continued at all points until checked by the weight of the concrete, which was placed in the vicinity of point 5 on October 27. Does not this mean that the area around point 5 might have risen considerably more than the 2.9 in. shown, before the rise was arrested by the weight of the concrete?

The amount and rapidity of the upward movement, prior to the placing of the concrete, leave the writer with a suspicion that a definite soil movement occurred due to lateral plastic flow induced by the weight of the surrounding unexcavated areas, causing uplift of the hard clay blanket. This thought is

supported by the fact (see discussion of Figs. 21 and 22) that "the street curbs moved in about $1\frac{1}{4}$ in. and down about $1\frac{1}{4}$ in.," with a certainty that the subsidence did not stop at the curbs but extended for some distance over the adjoining street areas. Moreover, it is noted that the total weight imposed on the soft clay at the underside of the hard clay blanket, outside the sheeted area, due to about 50 ft of overburden, was about 5,000 lb per sq ft, whereas the excavation within the sheeted area left a weight of only about 1,600 lb per sq ft to counteract uplifting pressure.

It would be of interest to know how the designers of the two Boston foundations applied live loads in their load-settlement computations. It is the practice in the writer's firm to determine as closely as possible, from the expected occupancy of a building, the live loads that will actually exist, and to use these

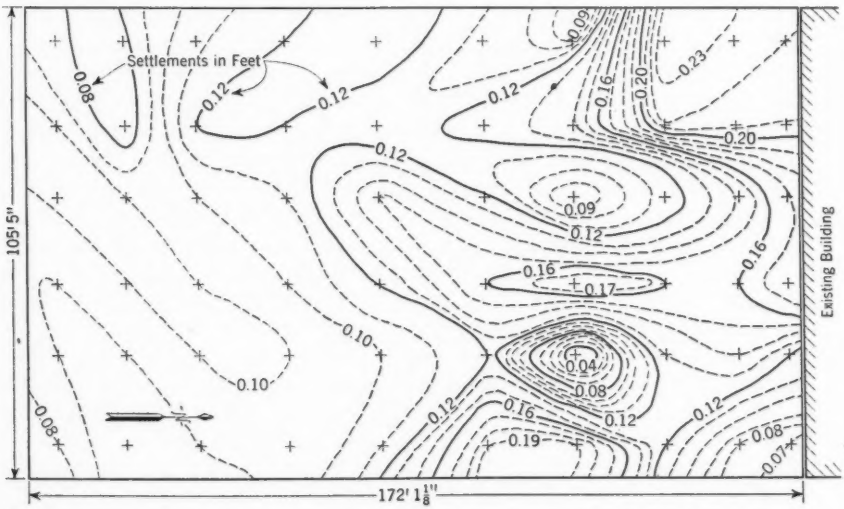

FIG. 31.—SETTLEMENT OF THE FOUNDATION OF THE ALBANY (N. Y.) MAIN TELEPHONE BUILDING AS OF DECEMBER 11, 1930

live loads, plus the actual dead loads, in load-settlement computations. As an example, in the design of the floating foundation for the Albany (N. Y.) Main Telephone Building,[66,67] the total live load assumed in settlement studies amounted to but one third of the total of the designed live loads for the several stories; this was believed to approximate the actual live load contributing to settlement.

It is unfortunate that so few settlement records are available, and that those available are so incomplete. The settlement of the foundations of the Albany Main Telephone Building, to December 11, 1930, is shown in Fig. 31. At that time practically all the live and dead loads were on the foundations, the dead load having been in place for practically a year. The south half of the mat

[66] "Rigid Rectangular Frame Foundation for Albany Telephone Building," by G. W. Glick, *Engineering News-Record*, November 27, 1930, pp. 836–838.

[67] *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Vol. 1, p. 278.

settled normally, and about as expected, with corners high and a tendency to dish downward at the center. The north half settled much more, and erratically, due to a cofferdam bracing failure at the north end of the site when the excavation was near subgrade level, more than 25 ft below the ground surface. This illustrates strikingly how the exigencies of construction may upset the calculations of the soil mechanic.

Fig. 32 shows settlement contours, as of February 9, 1927, for the 5-ft thick mat supporting the main office building of the Ohio Bell Telephone Company in Cleveland, Ohio. (The numbers in circles are the settlements, in sixteenths of an inch, extrapolated from records to December 3, 1926, or January 6, 1927.)

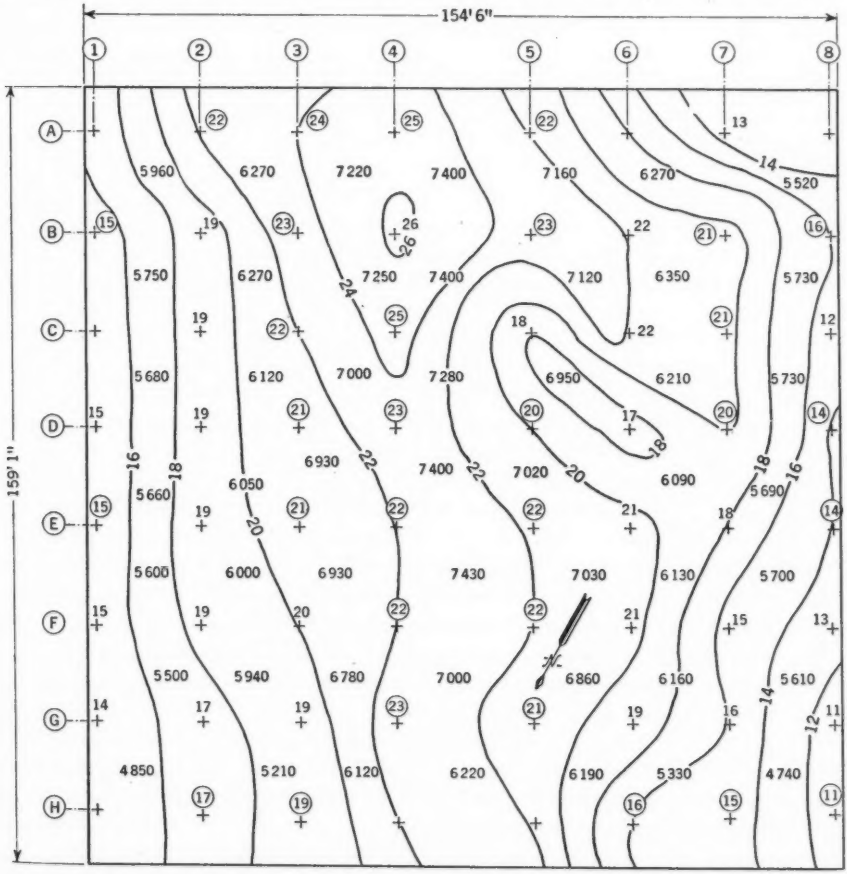

FIG. 32.—SETTLEMENT OF THE FOUNDATION, OHIO BELL TELEPHONE COMPANY BUILDING, AS OF FEBRUARY 9, 1927

This mat is nearly square, and is supported on a great depth of "stiff gray clay." The area had previously been loaded by a theater and a hospital. The bearing level is about 52 ft below the original ground surface, and the mat was designed for varying unit loads, the pressures on the soil being about 5,600 lb per sq ft

under the easterly and westerly side panels, and increasing to about 7,400 lb per sq ft in the north and south center panels. (To these loads should be added the uniformly distributed weight of a 5-ft thick concrete mat.) The average intensities of pressure on the soil, exclusive of the weight of the mat, are shown in each panel in Fig. 32, and it will be noted that the settlements followed rather consistently the variation in loading, resulting in a barrel-shaped deflection. The mat was completed in December, 1925, and all foundation work, including three levels of basement space, by March, 1926. Practically all of the dead weight was on the structure at the time of the settlements shown in Fig. 32.

With respect to statements (*a*) to (*m*) (see heading, "Stress-Deformation Characteristics of Cohesionless Soils"), concerning foundations supported on cohesionless soils, it would appear that the general principles outlined are based largely on theoretical considerations, and may or may not apply in practice. Foundations of structures of any magnitude or importance cannot be supported safely at the surface of a cohesionless soil, and are necessarily supported on bearing levels below the ground surface. Under actual conditions, rule (*a*) would not apply.

Rule (*b*) may or may not be applicable for deep caisson foundations. In the absence of data directly bearing on this point, the writer believes that uniform distribution of load over the bearing area is the only assumption that can be made. In the actual sinking of pneumatic and open-dredged caissons to great depths, so many factors upset any calculation of bearing pressures that much uncertainty exists as to the actual pressure distribution in the soil. The excavation in the area of a caisson shaft results in stress release over that area, and, to some extent, outside the excavated area. At the same time, the weight of the caisson, which is supported partly by skin friction and partly by direct bearing under the cutting edges, tends to set up high load intensities around the perimeter of the excavated area.

Subsequent filling and loading of the caissons results in some adjustment of supporting values of skin friction, cutting edge pressure, and direct bearing over the shaft area. In many cases, caissons sunk in granular soils may derive substantial support through side-wall friction, although such support is not generally allowed in the determination of safe bearing areas.

The writer questions the validity of rule (*d*)—that the ultimate bearing capacity of cohesionless soil increases as the square of the depth of the loaded area below the surface.

This rule seems to lead to fantastic results, if applied to very shallow foundations. The writer prefers, from present knowledge, to consider that the ultimate bearing value varies approximately with the depth, rather than with the square of the depth below the surface.

Although the general statements in (*e*) and (*f*) appear sound, the last statement in (*f*) may not be an infallible guide; certainly judgment and experience are required in its application. Settlements do follow the loading of cohesionless materials, and vary with the intensity of loading and density of the supporting soil. Over a large building area the density of the soil will usually vary considerably and differential settlements are to be expected. Load tests will

occasionally be advisable to check settlements, especially where high soil bearing intensities are used in the foundation design.

Rule (*j*) may not apply for deep caisson foundations where settlements may be due more to effect of construction operations than to soil characteristics. The writer's experience would indicate that settlement of a deep caisson foundation supported at a safe unit load on cohesionless soil is practically independent of the depth of the bearing level below the surface, and is largely dependent on soil disturbance and other factors connected with the construction procedure.

SOIL MECHANICS COMMITTEE OF THE LOS ANGELES SECTION—TRENT R. DAMES, CHAIRMAN,[68] FREDERICK J. CONVERSE,[69] AND STERLING S. GREEN,[70] ASSOC. MEMBERS, AM. SOC. E., AND PAUL BAUMANN,[71] M. AM. SOC. C. E.[71a]—The Soil Mechanics Committee of the Los Angeles (Calif.) Section first read the authors' paper individually and, surprisingly enough, the reaction of each individual was the same. Each felt that the authors had gone too far in publishing a series of brief generalizations regarding the behavior of sand and clay as the publication of generalizations of this nature, unsupported in the text by the data on which they were based and with the exceptions and limitations not stated, may mislead the uninitiated and lead to a loss of confidence on the part of, and the accusation of impracticability by, the construction industry.

When the committee met for detailed discussion of the paper, a new difficulty developed. No two members of the committee were in agreement concerning what the authors intended by some of their statements. For example, under the heading, "Soil Properties Responsible for Settlements," mention is made repeatedly of sand in the loose and dense states without qualification or definition. Sand usually is defined as consisting of more than 80% of particles ranging from 1/20 mm to 2 mm in grain size and the behavior of sands of different composition may be quite different. The division line between loose sand and dense sand presumably is the critical density; the latter, however, is not a soil constant but varies with the moisture content and loading condition and has been defined differently by different authorities. Another example of rather indefinite language is the reference to the "ultimate bearing capacity of a cohesionless soil." It is not clear whether the authors intend the ultimate bearing capacity to be the greatest load which it is permissible to place on a spread foundation or the load that will cause a footing to penetrate continuously through the soil in the manner of a pile being driven. There are several instances in which the authors state that one quantity varies in proportion to another quantity. In some of these cases it is not clear whether the authors are stating that the relationship is one of linear proportion or of some unstated curvilinear proportion.

---

[68] (Dames & Moore), Los Angeles, Calif.

[69] Associate Prof., Civ. Eng., California Inst. of Tech., Pasadena, Calif.

[70] Materials Engr., Bureau of Water Works and Supply, Los Angeles, Calif.

[71] Asst. Chf. Engr., Los Angeles County Flood Control Dist., Los Angeles, Calif.

[71a] Received by the Secretary August 2, 1943.

Apparently the authors intend the statement (*c*) (see heading, "Soil Properties Responsible for Settlements: Stress-Deformation Characteristics of Cohesionless Soils")—

> "The ultimate bearing capacity of a cohesionless soil increases approximately in direct proportion to the width of a loaded area"

—to be the basis for selecting design loads for different sizes of footings. This rule is valid only for a foundation on the ground surface. For footings founded at any depth, this rule does not apply even for the case of final failure. Furthermore, the yield point bearing capacity, which should be used for design purposes, will vary in proportion to the shearing strength of the supporting soil at the depth of the maximum concentration of stress. Statement (*d*)—

> "The ultimate bearing capacity of a cohesionless soil increases approximately as the square of the depth of the loaded area beneath adjacent ground surface"

is not borne out by the observations of the writers of this discussion. Certain very recent river-deposited sands in the Los Angeles area are capable of supporting, at their yield point bearing value, small spread foundations loaded to 5,000 lb per sq ft at a depth of 10 ft below the surface. Based on the authors' theory, the same deposits at a depth of 50 ft would support 125,000 lb per sq ft, and at 100 ft, 500,000 lb per sq ft. On the contrary, it would be expected that the yield point bearing capacity would vary as a straight-line function of the depth as the support offered a loaded area is a function of the resistance of the supporting material to shearing displacements by lateral or plastic flow. For a uniform soil condition such as the authors evidently are considering, the shearing strength increases approximately in direct proportion to the depth below the ground surface, and therefore it follows that the bearing value would increase approximately as a straight-line function of the depth.[72]

In paragraph (*f*) the statement is made that—

> "* * * for large areas on dense sand, or for areas which are to be founded at a considerable depth below ground surface on dense sand, even the heaviest loading which building codes permit is exceedingly conservative, so far as strength of the soil is concerned, and requires no verification by load tests."

The writers agree that, in general, building codes have restricted allowable bearing loads on dense sand to too low values. However, some advanced codes, such as the new Los Angeles City Building Code, permit deviation from arbitrary allowable foundation pressures after the performance of special foundation investigations. The only requirement of the Los Angeles Code is that "Stresses and deformations within the foundation shall be determined by the general principles of soil mechanics." If building codes impose costly and unnecessary restrictions on construction methods, the codes should be altered.

Statement (*g*)—

> "* * * the fact that Hooke's law does not apply to cohesionless soils also invalidates the law of superposition"

[72] "Practical Shear Tests for Foundation Design," by Trent R. Dames, *Civil Engineering*, December, 1940, pp. 784 and 785.

—should be expanded to say "Hooke's law does not apply to any soil." In all soils, the resistance to compression increases approximately as an exponential function of the load, similar to the behavior of metals under compressive stresses beyond the proportional limit.[73] However, the various equations for stress distribution, although certainly not precise, are valuable as an indication of the relative behavior of different sizes and types of foundations.

In paragraph (*l*) it is stated that for buildings founded on cohesionless soils, borings should be driven—

"* * * to a depth at least equal to the width of the building * * *."

Here again confusion and uncertainty result from the authors' failure to qualify the meaning of "building." For example, a typical airplane manufacturing building might be from 300 to 400 ft wide by 800 ft or more long with clear spans of 150 ft or more between rows of columns. It is unreasonable in this instance to require that borings extend to 300 or 400 ft in depth.

The authors devote several pages to a consideration of the characteristics of the consolidation diagrams of clay and particularly emphasize the determination of the preconsolidation loads without restricting the statements made to specific clays, such as the glacial clays of New England, or explaining the practical significance and use of the values obtained. The writers have not been able to apply the conclusions and rules cited to the residual and alluvial clays and bay muds of the Pacific Southwest as the consolidation curves of the clays of this region practically never possess straight-line parts on semi-logarithmic graphs. The consolidation curves obtained for the clays of this region in general approximate straight lines on double-logarithmic graphs.

In connection with Fig. 13, discussing "Strength of Clay and Safety Against Rupture," the authors state "* * * the possibility of stressing a cohesionless material to the point of rupture practically does not exist." Apparently the authors are referring to a dense, cohesionless soil far below the ground surface, although the limitations intended are not specified. Experience in the design, construction, and behavior of a variety of structures in the Pacific Southwest indicates that the possibility of overstressing a cohesionless material to the point of progressive failure is often the principal consideration. Within the capacity of the soil to resist lateral displacement by plastic flow, the deflection due to consolidation and compression of cohesionless soils (and also dense residual clays) is usually negligible.

In presenting Fig. 14, the authors make the statement that the shear strength of clay "* * * will be equal to a constant c * * *," regardless of the confining surcharge (normal pressure). If the clay is not completely saturated, increased granular pressures will be effective immediately upon application of load, and the shearing strength therefore will increase with the confinement and be expressed by the equation,

$$s = c + n \tan \phi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

[73] "Recent Work in the Field of High Pressures," by P. W. Bridgman, *American Scientist*, January, 1943, Fig. 15.

in which s is the total shearing strength; c, the apparent cohesion; n, the confining surcharge; and ϕ, the apparent angle of internal friction of the soil. For clays of this region, while the apparent angle of internal friction, ϕ, may be as low as 3° or 4°, it may also be as high as 20°.

In further discussing Fig. 14(*a*), the statement is made "* * * that the pressure q_0, which the footing must exert in order to produce failure * * * is equal to 4 c." By the use of another theory quoted, q_0 was equal to 5.5 c and again,[10] 5.14 c. Apparently, the authors are considering the conditions of final failure which is reached at considerably higher loads than those at which initial yielding occurs. The load which would produce this final failure would certainly not be a desirable basis for determining an allowable design load. A better appreciation of the behavior of loaded soils may be obtained through a study of the stresses required to produce yielding and plastic flow.[74,75] The yield point of a semi-infinite body loaded by a circular area on its surface is equal to π times the shearing strength. The ratios of unit load to shearing strength for other shapes of loaded areas do not vary appreciably from π. Practical confirmation of this relationship has been secured over a period of years.[72]

In the last few paragraphs under "Soil Properties Responsible for Settlements," the authors recommend the performance of cylindrical compression tests to determine the bearing capacity of clays. It is not clear whether the authors are referring to the use of triaxial compression tests or compression tests on unsupported soil cylinders. The authors, of course, realize that tests on unsupported soil cylinders are frequently impossible because of the character of the material. The direct shear test may be used effectively for all kinds of materials, without exception, and offers a means of determining bearing capacity with sufficient exactness for practical purposes at a cost which is comparable to the free cylinder test and which is much less than that of the time-consuming triaxial test.

The statement in the very last paragraph under "Soil Properties Responsible for Settlements" that, in the case of caisson foundations, "the hydrostatic uplift of the soil may be added to the foregoing value of 5 c and used as the ultimate bearing capacity" is very dangerous and subject to a long series of limitations most important of which is that the bearing capacity given by this rule is actually the capacity at ultimate failure and not a basis for allowable design loads.

The value of the second part of the paper, "Examples of the Application of Soil Mechanics to the Design of Building Foundations," would have been much enhanced if the authors had included information on the physical characteristics of the soils upon which the buildings in question are founded as this would enable the reader to make a comparison with similar soils, the behavior of which had been studied and experienced. The only such data, so far as the writers are aware, is that shown in Fig. 15 (*a*), which appears to be reminiscent of the usual well-driller's log except for the fact that it gives no record of ground

[10] Über die Eindringungsfestigkeit plastischer Baustoffe und die Festigkeit der Schneiden," by L. Prandtl, *Zeitschrift für angewandte Mathematik und Mechanik*, February, 1927.

[74] "Theory of Elasticity," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1934.

[75] "Plasticity," by A. Nádai, McGraw-Hill Book Co., Inc., New York, N. Y., 1931.

water. Above all, it is the clear and specific identification of the foundation soils which the writers (and probably many other readers interested and somewhat familiar with soil mechanics) hope to find in papers of this nature. Under the circumstances, this part of the paper, although definitely intriguing, is not as enlightening as it could be.

In closing, the writers hope that their discussion will be accepted as constructive criticism. They are well aware of the difficult, even impossible, task of setting down theories and principles of soil mechanics in the form of generalities, which have many exceptions, without the time or space for adequate discussion of the exceptions or clear statements regarding the limitations of the generalities.

Corrections for *Transactions:* In November, 1942, *Proceedings*, page 1509, Stuart Street end of Fig. 17, contour 1.00 to be included; and, on page 1517, line 4, after "Point 9" insert "(See Fig. 18(*a*))." See also corrections in the March, 1943, *Proceedings*, p. 454.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

STRENGTH OF SLENDER BEAMS

Discussion

BY MESSRS. T. MCLEAN JASPER, AND C. M. GOODRICH

T. MCLEAN JASPER,[4] M. AM. SOC. C. E.[4a]—A mathematical attempt to solve a very important problem, and one which is recognized as a necessary addition to the problems dealing with buckling, is very welcome. As a supplement to the work by Professor Winter, the writer would cite the tests on light I-beams by Jasper O. Draffin, M. Am. Soc. C. E., and Milo S. Ketchum, Jr., Assoc. M. Am. Soc. C. E., in 1932 (15)[4b] and those by Harald M. Westergaard, M. Am. Soc. C. E., in 1922 (16). The purpose of presenting the first of these references is to suggest some test results to which the author may connect his theories, and the second reference to the generalized problem of buckling which the author has not mentioned should be a helpful extension to the Bibliography.

The writer is particularly intrigued by Fig. 2 and the application of mechanics to this problem. This paper offers a method of approach that can help the designer to simplify the application of the problem. The problem is not a simple one. The physical properties of the material used influence the heavier beams greatly whereas the $E\,I$-values of the shape and material undoubtedly influence the lighter sections.

The other references (17)(18)(19), with their respective discussions, present very similar cases and may be useful to students of this problem in supplying test data and analysis methods that may inspire further tests which apply more directly to safe designs for slender beams.

C. M. GOODRICH,[5] M. AM. SOC. C. E.[5a]—In the usual case of the loading in Fig. 6, an eyebeam ideally straight would be prevented from twisting about its longitudinal axis. In one with a slight turn, twisting would be permitted only up to the point where there was flat contact between the two pieces.

NOTE.—This paper by George Winter, Esq., was published in June, 1943, *Proceedings*.

[4] Research Dept., A. O. Smith Corp., Milwaukee, Wis.

[4a] Received by the Secretary June 25, 1943.

[4b] Numerals in parentheses, thus: (15), refer to the corresponding items in the Bibliography (Appendix I in the paper), and at the end of discussion in this issue.

[5] Cons. Engr., The Canadian Bridge Co., Ltd., Walkerville, Ont., Canada.

[5a] Received by the Secretary July 9, 1943.

In the case of columns, two eccentricities are considered, the first due to an eccentric end load, and the second due to the maximum deviation of the axis from a straight line between the ends. Only the latter needs to be considered in the case of beams.

FIG. 6

If the rule for reduction of stress in beams, either in the American Institute of Steel Construction (A.I.S.C.) specifications or in those of the Canadian National Building Code, is multiplied by $\frac{10}{6}$, a result fairly close to $40{,}000 - \frac{100\,L}{i}$ is obtained (assuming $i = 0.2\,b$). The assumption that $i = 0.2\,b$ was not too violent when only American standard beams were rolled. For this assumption, there is now a variation of from $i = 0.15\,b$ to $i = 0.27\,b$ for rolled beams, with grotesquely varying results if the moment of inertia is accepted as a measure of stiffness; and, of course, built girders and box beams are not included in the rule, which leaves these cases in the air.

Brik (20) presents eight tests, including those for built and box girders; H. F. Moore (21) gives forty tests, reviews many more, and recommends the formula,

$$\sigma = 40{,}000 - \frac{m\,l}{i} \qquad (71)$$

in which $m = 1$ for a cantilever, $m = \frac{2}{3}$ for a uniformly loaded beam, etc. In any case of two supports, the constant m is the ratio of the area of the moment diagram to its enclosing rectangle. The range of variation in the tests is stated to be $\pm$ 2,500 lb per sq in. His testing arrangement gives the test beam every opportunity to twist if so inclined. Assuming $i = 0.2\,b$, for $200\,\frac{L}{i} = 40\,\frac{L}{b}$, the minimum stress would be more than 29,000 lb per sq in. for uniformly loaded beams against about 17,000 in Fig. 4. At $\frac{L}{i} = 0$, Professor Moore gives 37,500 from tests, while Fig. 4 gives 29,000. The author cautions that (see heading, "Beam Curves") "The proper degree of imperfection, K, to be used in design can only be established by extensive tests." It is of interest, however, to note what existing tests reveal, and apparently they show that the A.I.S.C. and Canadian National Building Code formulas are undesirable, while the author's mathematics yield no rule for the designer.

The factor of stiffness is taken by S. Timoshenko (22) as $\frac{A^4}{40\,(I_{xx} + I_{yy})}$, and, in his example of a 24-in., 79.9-lb I-beam, it is 3.46. This is the Saint Venant rule, which, however, was intended for sections with a chunky solid center, and not for such shapes as I-beams. For the same section the Bethlehem handbook gives 5.01 as the factor of stiffness, and $\Sigma\,\frac{1}{3}\,w\,t^3$ equals about 4, if average thickness is used in the flanges. (August Föppl's tests (23) led him to the factor of stiffness of 1.29 for this type of section, and 1.29×4 = say 5.1.)

In designing, when unhampered by specifications, one may perhaps use Professor Moore's rule up to $200\frac{L}{i}$, and, for greater values, some such formula as

$$\sigma = \frac{1{,}280{,}000{,}000}{\left(\frac{l}{i}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (72)$$

for uniformly distributed loads, remembering that about 2,500 lb per sq in. is to be subtracted on account of the variation in beams.

The result, of course, is merely a probable ultimate for a condition to which any actual case is likely to be an approximation.

Bibliography.—

(15) *Bulletin No. 241*, Eng. Experiment Station, Univ. of Illinois, Urbana, 1932.

(16) **"Buckling of Elastic Structures,"** by H. M. Westergaard, *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 576.

(17) **"Strength of Steel Columns,"** by H. M. Westergaard and William R. Osgood, *Transactions*, A. S. M. E., Vol. 50, No. 17, 1928, p. 65.

(18) **"The Collapse Strength of Steel Tubes,"** by T. M. Jasper and J. W. W. Sullivan, *Transactions*, A. S. M. E., Vol. 53, 1931, p. 219.

(19) **"Importance of Compression Yield Strength in Established Setting Depths for Casing,"** by T. McLean Jasper, *Drilling and Production Practice*, Am. Petroleum Inst., 1936.

(20) *Der Eisenbau*, 1912, p. 351.

(21) **"The Strength of I-Beams in Flexure,"** *Bulletin No. 68*, Univ. of Illinois, Urbana, 1913.

(22) **"Beams Without Lateral Support,"** by S. Timoshenko, *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 1247.

(23) *Sitzungberichte d. bayer, Acad. d. Wiss.*, S. 295, 1921; Versuche u. d. Verdrehungsfestigkeit d. Walzeisenträger.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE HYDRAULIC JUMP IN SLOPING CHANNELS

Discussion

BY MESSRS. FRANK S. BAILEY, AND G. H. HICKOX

FRANK S. BAILEY,[26] Assoc. M. Am. Soc. C. E.[26a]—Paraphrasing the Declaration of Independence, all men are endowed with certain rights, and among them are life, liberty, and the pursuit of knowledge.

The pursuit of knowledge often involves much labor and, when knowledge is attained by any one engaged in research and is presented for the consideration of other researchers, some advance in the understanding of the particular topic under consideration is almost certain to follow.

Many members of the engineering profession are engaged on work which must be pushed forward with as little delay as is consistent with reasonable accuracy. Thus, when a job is under way, they often must rely for basic information on the analyses and experimentation of others. The writer realizes the complexities and difficulties of the subject the author has undertaken to clarify and trusts that the paper will be of great value in its field.

G. H. HICKOX,[27] M. Am. Soc. C. E.[27a]—The hydraulic jump as a means of dissipating energy has been known and studied for many years. The underlying theory is relatively simple, and has been stated clearly by Mr. Woodward.[28] A number of studies of its performance on a horizontal surface have been made, and the agreement of experiment with theory is gratifying, particularly with respect to the vertical dimensions. In many practical applications it is desirable, and sometimes necessary, to produce the jump on a sloping surface rather than on a horizontal one. The prediction of the final depth for this condition has not been entirely satisfactory heretofore. The phenomenon is governed by the same laws as when it occurs on a horizontal

NOTE.—This paper by Carl E. Kindsvater, Jun. Am. Soc. C. E., was published in November, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1943, by Messrs. Joe W. Johnson, and Karl R. Kennison; May, 1943, by Messrs. J. C. Stevens, and C. J. Posey; and June, 1943, by Jerome Fee, Assoc. M. Am. Soc. C. E.

[26] With Stone & Webster Eng. Corp., Boston, Mass.

[26a] Received by the Secretary June 16, 1943.

[27] Senior Hydr. Engr., TVA, Hydraulic Laboratory, Norris, Tenn.

[27a] Received by the Secretary July 28, 1943.

[28] "Theory of the Hydraulic Jump and Backwater Curves," by Sherman M. Woodward, *Technical Reports*, The Miami Conservancy Dist., Pt. III.

surface, but trouble has been experienced in applying them. The difficulty occurs in determining the horizontal component of pressure exerted on the mass of water within the jump proper by the sloping bottom.

Mr. Kindsvater presents an ingenious and rational method of accounting for the pressures on the sloping surface. The fact that his analysis includes an experimentally determined coefficient ϕ does not detract from its value. The paper was the outgrowth of studies, made at the Hydraulic Laboratory of the Tennessee Valley Authority under the direction of the writer, to establish the elements of the jump on sloping surfaces. The method of attack was suggested by an earlier paper by Messrs. Bakhmeteff and Matzke[29] in which it was shown that the horizontal and vertical elements of the jump could be expressed as functions of λ.

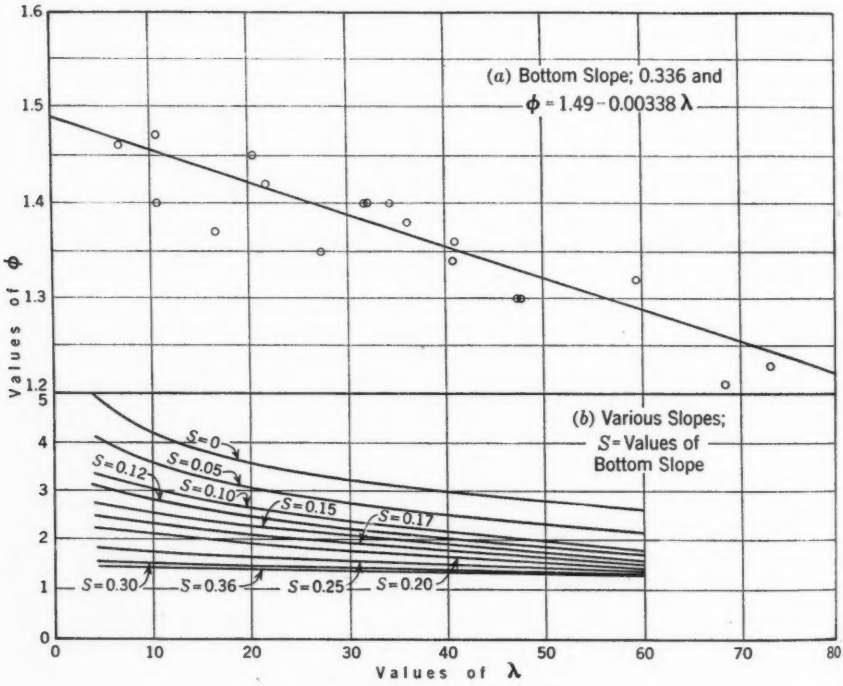

FIG. 18.—VALUES OF ϕ AS A FUNCTION OF λ

This discussion presents the results of later experiments made on a slope of 1 on 3 and shows that the author's analysis is applicable to any slope on which a satisfactory jump can be formed. The tests were made in a flume 3.5 ft wide with discharges varying from 0.760 to 3.966 cu ft per sec, initial velocities from 5.57 to 9.68 ft per sec, and initial depths from 0.028 to 0.179 ft. The values of λ ranged between 6.9 and 73.0. Fig. 18(*a*) shows values of ϕ as a function of λ for these tests.

[29] "The Hydraulic Jump in Terms of Dynamic Similarity," by B. A. Bakhmeteff and A. E. Matzke, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), pp. 630–647.

Fig. 18(*b*) has been prepared to show the relationship between ϕ and λ for slopes between the horizontal and 1 on 3. Data for the horizontal slope have been taken from the paper by Messrs. Bakhmeteff and Matzke.[29] The curve for $S = 0.336$ is the same as that in Fig. 18(*a*). The other curves have been interpolated using the Kindsvater data as a guide. With the value of ϕ given by Fig. 18, it is possible to calculate d_2 for any slope by means of Eq. 14.

Calculation of Depth d_2.—In calculating the depth d_2 the writer prefers to use Eq. 24 which is dimensionless and therefore directly applicable to a large range of variables. When the relationship between ϕ and λ is known for a

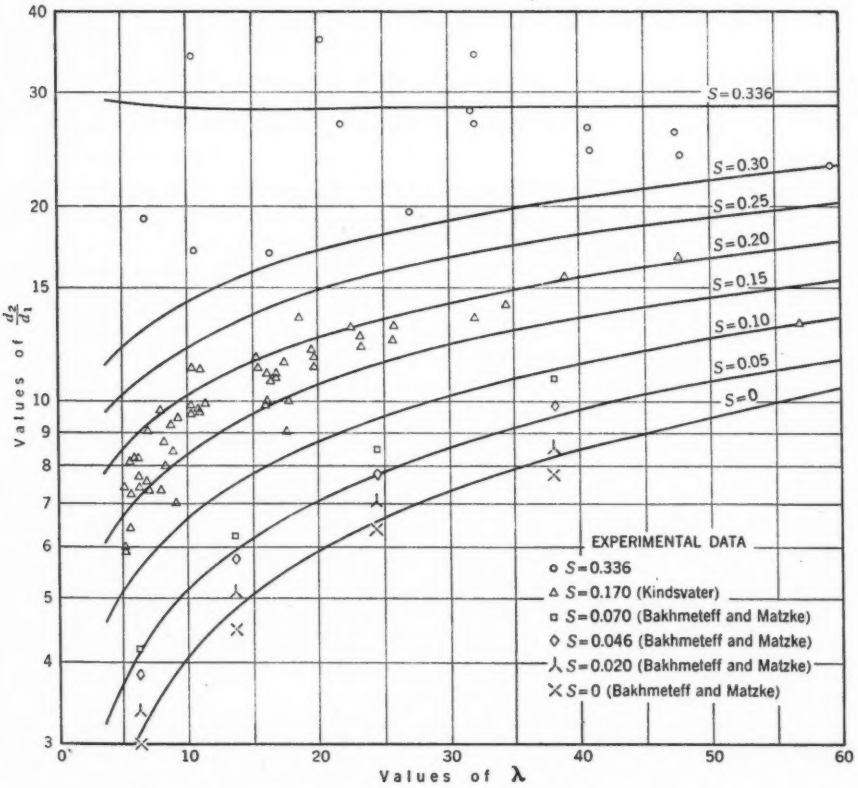

FIG. 19.—RELATION BETWEEN λ AND $\frac{d_2}{d_1}$

given slope, $\frac{d_2}{d_1}$ can be expressed as a function of λ for that slope. It has been calculated for slopes between 0 and 0.336 for values of λ between 5 and 60 using the curves of Fig. 18(*b*), and the results are shown in Fig. 19. The experimental values of $\frac{d_2}{d_1}$ found for the 0.17 slope by Mr. Kindsvater and for the 0.336 slope by the writer also are shown in Fig. 19 for comparison.

A paper by Messrs. Bakhmeteff and Matzke[30] includes data on four flat slopes, including the horizontal, from which the ratio $\frac{d_2}{d_1}$ can be calculated directly from experimental results. The points thus determined also are plotted in Fig. 19. In general, they agree reasonably well with the interpolated curves. It may be noted that the points for the horizontal surface fall slightly below the corresponding curve, indicating the possibility of experimental error. If the points for the other slopes are similarly in error, the agreement with the curves is improved accordingly. In general, the agreement between the calculated curves and the experimental results is good and it is believed that the curves are satisfactory for practical estimates of the jump height.

Location of End of Jump.—The location of the end of the jump is a matter on which there seems to be little agreement among investigators. The precise point to be chosen is open to question. Some authorities assert that the end of the jump is marked by the highest water-surface elevation. This is not true on a slope, however, since a part of the rise is not due to the jump itself but is caused by the conversion of kinetic to potential energy that occurs in any expanding channel. Hence, the highest elevation has no particular significance with respect to the jump. In addition, it is practically impossible to determine a corresponding point in observations of field installations because of the extreme surface turbulence and also because the recovery of velocity head varies considerably due to local variations in channel shape. Another criterion for determining the length of the jump is the distance to the point at which the bottom velocities are no longer dangerous to the bed. This distance is not a satisfactory measure as it depends on the resistance of the bed to erosion and has no absolute basis for determination. It also is extremely difficult to determine in a prototype structure. The location chosen by the author (the end of the roller) is one that can be determined with comparative ease both in the laboratory and in the field and is preferred by the writer for this reason. It is open to the objection that on steep slopes seriously high bottom velocities may exist beyond this point, and thus it does not necessarily indicate the length within which the high-velocity jet loses its destructive energy. On a horizontal surface the end of the roller is some distance from the end of the jump but on slopes as steep as 1 on 6 and steeper, the elevation at the end of the roller is practically equal to the highest water surface.

In practical applications of the hydraulic jump to the dissipation of energy, these differences in location of the end of the jump become unimportant. In most cases it is not economical to provide protective paving on the river bottom for the full length of the jump as defined by the highest water surface. It is frequently much cheaper and equally satisfactory to stop the apron at a point some distance upstream and to terminate it with a low sill having a sloping upstream face. Such a sill deflects upward from the river bed any high-velocity flows that still might exist. It also extends the range of depths over which a satisfactory jump will occur by maintaining a sufficient depth to produce the jump if for some reason the tailwater should fall below its expected

[30] "The Hydraulic Jump in Sloped Channels," by B. A. Bakhmeteff and A. E. Matzke, *Transactions*, A.S.M.E., Vol. 60 (1938), pp. 111–118.

elevation. An extended discussion as to exactly what constitutes the end of the jump is thus largely academic. The length of paving below a spillway is subject to economic considerations and seldom is determined finally by the theoretical length of the jump. Further discussion of this subject properly belongs to a study of stilling-basin design.

In the preparation of Figs. 18(*b*) and 19 two definitions of the end of the jump have been used. Values of ϕ for $S = 0$ in Fig. 18(*b*) are from data by Messrs. Bakhmeteff and Matzke[29] for which the end of the jump was taken at the highest tailwater elevation. For the Kindsvater data on the 0.17 slope and the writer's data on the 0.336 slope, the end of the jump was taken at the end of the roller. As stated, for slopes steeper than 1 on 6, the two elevations are practically identical. For the level floor, the highest elevation of the water surface is preferable since the elevation at the end of the roller is somewhat lower. It is believed that interpolation between the values used for the level floor and the 0.17 slope will give satisfactory results.

Calculation of Bottom Elevation.—The calculation of the depth d_2 at the end of the jump on a sloping surface has no meaning unless the depth can be measured at a known point along the slope. Messrs. Bakhmeteff and Matzke[29]

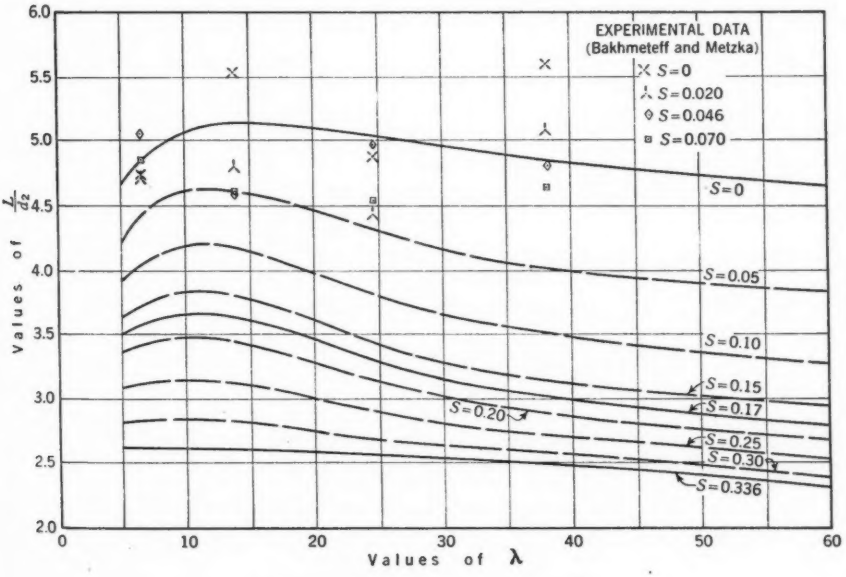

FIG. 20.—RELATION BETWEEN λ AND $\frac{L}{d_2}$

have shown that the ratio $\frac{L}{d_2}$ is a function of λ for a jump on a horizontal surface. The curve for $S = 0$ in Fig. 20 is taken from their data. The curve for $S = 0.17$ is reproduced from Fig. 6(*b*), and the curve for $S = 0.336$ is based on the experimental data on the 1-on-3 slope. Messrs. Bakhmeteff and Matzke considered the length L to be the distance to the highest water surface and the

curve for $S = 0$ is constructed accordingly. Mr. Kindsvater, on the other hand, measured L to the end of the roller. The curves are thus inconsistent in this respect. However, for a 1-on-6 slope, the end of the roller and the highest water surface are practically at the same elevation. The broken lines of Fig. 20 represent an attempt to interpolate tentative curves that may be useful for design purposes. The plotted points represent the results of experiments by Messrs. Bakhmeteff and Matzke on relatively flat slopes. The agreement is not perfect but indicates that the curves are approximately correct. Fig. 20 is sufficiently accurate for practical design purposes.

It is frequently necessary to determine the required bottom elevation to insure the formation of the jump with a given discharge and tailwater elevation. This can be done by assuming an elevation slightly below tailwater as the point of beginning of the jump. Having determined d_1 and λ for this point, the depth d_2 and the length L are determined in that order with the aid of Figs. 19 and 20, respectively. The elevation of the bottom at distance L from the beginning then is found by dividing L by the slope ($\tan \alpha$) and subtracting the result from the bottom elevation at the assumed point of beginning. The difference between tailwater elevation and the bottom elevation thus calculated should be equal to d_2. If the difference is smaller than d_2, another trial must be made assuming a lower point of beginning. Only a very few trials are needed to determine the necessary dimensions with sufficient accuracy.

Case 2 Jump.—Data on the 0.336 slope for the Case 2 jump were not obtained. It was not possible, therefore, to supply a curve similar to Fig. 8 for other slopes.

Effect of Approximate Initial Pressure on Final Depth.—The effect of the approximation to the initial pressure (Eq. 9) on the final depth may be determined to a first approximation by assuming that the neglected difference in initial pressures affects the final depth so little that the change in final velocity may be neglected. The error in initial pressure, ΔP_1, is the difference between the approximate pressure and the correct pressure, or

$$\Delta P_1 = \frac{w\,(d_1)^2}{2\cos^2\alpha} - \frac{w\,(d_1)^2}{2} = \frac{w\,(d_1)^2}{2}\left(\frac{1}{\cos^2\alpha} - 1\right) = \frac{w\,(d_1)^2}{2}\tan^2\alpha \quad (56)$$

Neglecting corrections to the momentum at section 2, the error in pressure P_2 is equal to the error in pressure P_1. The error at section 2 is

$$\Delta P_2 = \frac{w\,(d_2 + \Delta d_2)^2}{2} - \frac{w\,(d_2)^2}{2} = \frac{w}{2}\left[2\,d_2\,\Delta d_2 + \Delta(d_2)^2\right] \quad (57)$$

Neglecting the square of Δd_2,

$$\Delta P_2 = w\,d_2\,\Delta d_2 \quad (58)$$

Equating 56 and 58, $\frac{w\,(d_1)^2}{2}\tan^2\alpha = w\,d_2\,\Delta d_2$; from which

$$\Delta d_2 = \frac{(d_1)^2}{2\,d_2}\tan^2\alpha \quad (59)$$

It is more convenient to write the ratio of the error in final depth to the depth:

$$\frac{\Delta d_2}{d_2} = \left(\frac{d_1}{d_2}\right)^2 \frac{\tan^2 \alpha}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (60)$$

The error for a slope of 1 on 3 and a depth ratio of 0.4 is 0.9%. For smaller depth ratios and flatter slopes the error is less. For practical purposes it may be neglected.

The actual pressure at section 1 seems to be rather uncertain. The pressures on both sides of a vertical plane at that point should be equal. Approaching the section from upstream, the pressure on the section for uniform parallel flow is given by Eq. 8. On the downstream side of the section, experiment shows that the bottom pressure is $\frac{w\, d_1}{\cos \alpha}$. Since the vertical depth is $\frac{d_1}{\cos \alpha}$, the total pressure on the section is $\frac{w\,(d_1)^2}{2 \cos^2 \alpha}$ which is Eq. 9. This indicates a discontinuity of pressure at section 1 or, more probably, a rapid change in the type of flow. The use of Eq. 9 thus may be justified on other grounds than expediency.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SEDIMENTATION IN RESERVOIRS

Discussion

BY JOE W. JOHNSON

JOE W. JOHNSON,[4] ASSOC. M. AM. SOC. C. E.[4a]—An excellent discussion of the origin, transportation, and deposition of the sediments in reservoirs has been presented in this paper. Of interest to the writer is the section of the paper concerning the transportation of sediment to the reservoir. Some of the phases of this problem well may be extended in the light of recent investigations.

For practical considerations the total sediment load of a stream consists of three parts—the wash load (6),[4b] the coarse material in suspension, and the coarse material rolling or sliding along the bed. The equipment and method of calculation used in estimating these loads differ greatly. The wash load may be estimated very simply by continuous and frequent sampling during stream rises. In some instances, the use of distribution graphs of suspended-matter concentration (33) or turbidity data (34) may greatly simplify the labor of determining the load of wash material. The wash load is composed of relatively fine material and constitutes much the greater part of the material in a suspended-load sample. It may be regarded as sediment in relatively permanent suspension. The settling velocity of the wash material is very small, and, once the material is washed into a stream, it is transported completely through the stream system, by even the most feeble velocities, to the point of deposition. It is the wash material which probably constitutes the bulk of sediment that passes through reservoirs in muddy underflows. The quantity of wash material transported in a stream bears no relation to the discharge because the conditions of supply, which vary from year to year and season to season, are independent of flow conditions.

The author states (see heading, "Transportation of Sediment to the Reservoir: Bed Load") that "bed load, unlike suspended load, is directly related to the discharge at all times, because the sediment supply for the bed-load forces

NOTE.—This paper by Berard J. Witzig, Jun. Am. Soc. C. E., was published in June, 1943, *Proceedings*.

[4] Asst. Prof., Dept. of Mech. Eng., Univ. of California, Berkeley, Calif.

[4a] Received by the Secretary July 19, 1943.

[4b] Numerals in parentheses, thus: (6), refer to corresponding items in the Bibliography (see Appendix II in the paper), and at the end of discussion in this issue.

is nearly always available * * *." This statement which infers that no direct relationship exists between suspended load and discharge is true only when total suspended load is considered, but needs qualification when the loads of sediment of various size classifications are considered separately. The suspended load, including both fine and coarse material, is not directly related to the discharge because of the variability of the wash load. The coarse material in suspension, however, is related to the discharge because the supply of this material is always available, except, as stated by the author (see same heading), "on debris-clear rock-lined channels." An excellent illustration, showing how the coarser grades of sediment are related to discharge, is given by the extensive measurements of C. P. Vetter, M. Am. Soc. C. E. (35).

The author suggests the use of "silt-rating curves" in estimating the suspended load of streams. In such a practice, of course, it is assumed that the suspended load is directly related to discharge. The curves given by the author, along with the data of Messrs. Campbell and Bauder (16), indicate that an approximate relationship may be developed between instantaneous values of discharge and suspended load, presumably including both fine and coarse material. In these instances, the period of observation has extended over only two or three years when conditions of supply may have been approximately constant. In connection with the extensive sediment investigations on the Missouri River and its tributaries by the U. S. Engineer Office, silt rating curves were plotted for twenty-two stations (36). Except for the curve of the Kansas City station on the Missouri River, which has been published elsewhere (37), the curves for the various stations have not been published. These curves may be examined, however, in the office of the Chief of Engineers in Washington, D. C. Some of these curves show a usable relationship, as found for the Kansas City station, whereas other plots show such a large scattering of data that their use is of questionable value. As additional data to the information given in Table 4, the equation of the Kansas City curve reported by L. G. Straub, M. Am. Soc. C. E. (37), is

$$G_s = 30.42 \times 10^{-11} Q^{2.16} \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

The minimum discharge was 20,000 cu ft per sec and the maximum discharge was approximately 187,000 cu ft per sec.

For streams on which observations, extending over a period of several years, are available, a silt rating curve based on annual rates of runoff and sediment load shows some scattering of points but appears to give a more simple relationship for estimating the average annual suspended load. For example, Fig. 4 shows a plot of annual runoff and suspended load for the Colorado River and its tributaries as observed by the U. S. Geological Survey (38). These data were obtained for periods from twelve to sixteen years. With the average annual runoff being estimated from long-term discharge records from the various streams, these curves can be entered directly to obtain the approximate average annual suspended load.

Although an estimate of the average annual contribution of suspended load of a stream requires considerable field and office work, an estimate of the average annual contribution of bed load can be made accurately and con-

veniently from the application of a suitable bed-load formula and a flow-duration curve. The author has suggested the Schoklitsch and the M.I.T. bed-load formulas, neither one of which has had the extent of verification in natural streams that the $(\phi - \psi)$-relationship (21) has had. The formulas

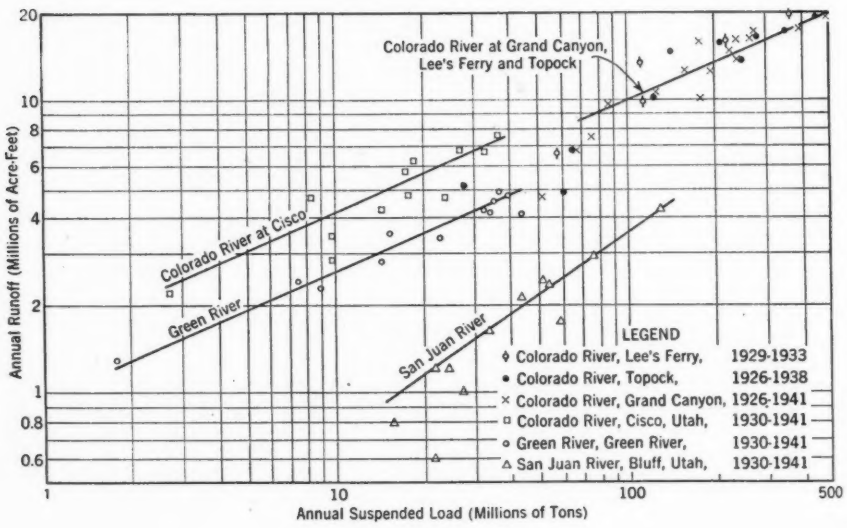

FIG. 4

expressed by Eqs. 5*a* and 5*b* include a "critical" discharge to describe the condition when movement of the bed material begins. "Unfortunately, this is a condition that does not exist in nature" (21*a*). In many instances, the magnitude of the critical discharge is negligible in comparison with the discharge under consideration and might just as well be discarded.

The author states (paragraph preceding heading, "The Action of Sediment in Reservoirs") that the Einstein functions "are quite complex, and do not readily permit estimates of bed load for the varied conditions met in the field." Admittedly, the functions are quite formidable in appearance on first examination. However, further consideration will show that many of the terms are constants, and calculations of the rate of bed-load transportation in a particular reach of a stream can be made quickly and conveniently from a few simply observed field data.

In the Einstein formula the intensity of transportation is given by the expression

$$\phi = \frac{q_b}{F(s_s - s_f)\,\gamma}\sqrt{\frac{s_f}{s_s - s_f}}\,\frac{1}{g^{0.5}\,D^{1.5}} \qquad (9a)$$

and is related to the hydraulic variable,

$$\psi = \frac{s_s - s_f}{s_f}\,\frac{D}{S_e R} \qquad (9b)$$

in which q_b is the rate of bed-load transportation in weight under water per unit width per second; F is a dimensionless factor in the Ruby equation for settling velocity and is a function of the representative grain diameter D; g is the acceleration of gravity; s_s and s_f are the specific gravity of the solid and fluid, respectively; γ is the unit weight of the fluid; S_e is the slope of the energy grade line; and R is the hydraulic radius of the bed.

For very wide channels—that is, where the energy dissipated along the banks is negligible compared to the energy dissipated along the bed—the ϕ-terms and ψ-terms can be used as follows to construct a set of curves or to prepare tables that show the rate of bed-load transportation per unit width as a function of either discharge or depth for various slopes and bed material. Thus, if $s_s = 2.65$, $s_f = 1$, and $\gamma = 62.3$ lb per cu ft, the ϕ-terms and ψ-terms may be written as follows:

$$\phi = \frac{q_b}{F\,D^{1.5}\,(1.65)^{1.5}\,(62.3)\,(32.2)^{0.5}} = \frac{q_b}{750\,F\,D^{1.5}} \quad \ldots\ldots (10a)$$

and

$$\psi = \frac{1.65\,D}{S_e\,d} \quad \ldots\ldots (10b)$$

in which d is the depth of flow. For a certain representative grain diameter (that is, the grain size at which 35% of the mixture is finer) and channel slope, values of ψ can be computed for various assumed depths of flow. From the Einstein curve showing the relation between ϕ and ψ, the value of ϕ corresponding to the computed value of ψ is obtained and the value of q_b is then computed. The total load, G_b, is $q_b B$ in which B is the channel width. A curve showing the relationship between G_b and depth of flow (or Q) may then be constructed. In the event that the friction along the side-walls cannot be neglected, the hydraulic radius of the bed should be used in place of the depth in the equation for ψ.

Field measurements by Mr. Einstein in 1941–1942 (39)(40) made on the rates of bed-load transportation in Mountain Creek near Greenville, S. C., and West Goose Creek near Oxford, Miss., using a portable measuring apparatus, have verified the $(\phi - \psi)$-relation previously developed from flume studies and have extended the range of application to $\phi = 10$. This range of application includes small streams and canals with fine sediment or large streams with coarse sediment.

A determination of the average annual volume of bed load transported by a stream may be obtained by using the relationship between discharge and rate of transportation in conjunction with the flow-duration curve of the stream (41). A table showing the frequency of occurrence of various discharges is prepared from the duration curve—that is, the total range in discharge is divided into a convenient number of parts and the percentage of time that the mean discharge in each part prevails is determined and tabulated. The number of seconds that each discharge prevails per year is computed. The amount of bed load that is transported per hour for each discharge may be determined from the previously established relationship between discharge and rate of transportation. The rate of transportation for each discharge,

multiplied by the total number of seconds that the particular discharge prevails per year, gives the annual amount of bed load for that discharge. The sum of the amounts of bed load for the various discharges gives the average annual weight of sediment transported as bed load. This annual load on a weight basis can be converted to volume by merely multiplying by the weight per cubic foot of the material.

On streams where discharge records are not available for the construction of a flow-duration curve, the duration curve for a neighboring stream (corrected for size of drainage basin) may be used because investigations have shown that the flow-duration curves are similar for various streams in a locality where topography, climate, and vegetation are similar.

Bibliography.—

(6) **"A Distinction Between Bed-Load and Suspended Load in Natural Streams,"** by H. A. Einstein, Alvin G. Anderson, and Joe W. Johnson, *Transactions*, Am. Geophysical Union, 1940, pp. 628–633.

(16) **"A Rating Curve for Determining the Silt Discharge of Streams,"** by F. B. Campbell and H. A. Bauder, *ibid.*, pp. 603–607.

(21) **"Formulas for the Transportation of Bed Load,"** by H. A. Einstein, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), pp. 561–577. (*a*) p. 595.

(33) **"Distribution Graphs of Suspended-Matter Concentration,"** by Joe W. Johnson, *ibid.*, Vol. 108 (1943), p. 941.

(34) **"Use of Turbidity Determinations in Estimating the Suspended Load of Natural Streams,"** by R. G. Grassy, *Journal*, Am. Water Works Assn., Vol. 35, No. 4, April, 1943, pp. 439–453.

(35) **"Why Desilting Works for the All-American Canal?"** by C. P. Vetter, *Engineering News-Record*, Vol. 118, No. 8, 1937, p. 321.

(36) **"Missouri River,"** *H. R. Doc. No. 238*, 73d Cong., 2d Session, 1935, p. 1056.

(37) **"Transportation of Sediment in Suspension,"** by L. G. Straub, *Civil Engineering*, May, 1936, p. 321.

(38) **"High Silt Content of Colorado River,"** *Engineering News-Record*, Vol. 130, No. 14, April 8, 1943, pp. 502–503.

(39) **"The Flow of Water on a Movable Bed,"** by H. A. Einstein, 2d Hydraulic Conference, Univ. of Iowa, Iowa City, 1942.

(40) **"Bed-Load Transportation in Mountain Creek,"** by H. A. Einstein, Office of Research, SCS, U.S.D.A., 1943.

(41) Discussion by Joe W. Johnson of **"Statistical Analysis of Hydrology,"** by L. R. Beard, *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1130.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ORGANIZING AND FINANCING SEWAGE TREATMENT PROJECTS

Discussion

BY A. BRINTON CARSON

A. BRINTON CARSON,[7] ASSOC. M. AM. SOC. C. E.[7a]—In his excellent paper Mr. Greeley states that the trend in the organization of sewage treatment projects has been independent of other local governments; but it appears, from his paper, that the trend proceeds at a snail's pace. It also appears that the existence of such a trend at all is due, not to the practical considerations of engineering, but to the legal exigencies of financing.

The evils of organizing sewage treatment projects along purely municipal or local governmental lines have always been apparent; so too have the engineering fallacies to which this practice frequently leads. Mr. Greeley has indicated, either consciously or unconsciously, the difficulties encountered in large municipalities. Those encountered in small communities are frequently defined more sharply.

The most important of these difficulties is the problem of operation. Because small plants frequently discharge their effluent into small streams, the required degree of treatment is generally higher than for large plants discharging into larger bodies of water. Yet it is precisely for small plants that the greatest difficulty is encountered in securing properly trained operators. The difficulty, of course, is basically financial. The proper operation of small plants tends to increase the per capita cost of operation, sometimes seriously. It is apparent, therefore, that larger plants, or groups of small ones operated as a unit, could operate more efficiently and at a much reduced per capita cost.

There are other difficulties in small communities, however. Consider the case of C and D, two small adjoining towns on the same watershed. Because they were unable to agree on a joint plan, each built its own treatment plant. Both plants were finally constructed 200 yd apart on the same bank of one

NOTE.—This paper by Samuel A. Greeley, M. Am. Soc. C. E., was published in December, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1942, by Messrs. Milton P. Adams, and Hal F. Smith; April, 1943, by Herbert Moore, Assoc. M. Am. Soc. C. E.; and May, 1943, by Francis H. Kingsbury, M. Am. Soc. C. E.

[7] Lt., CEC-V(S), U. S. N. R., Philadelphia, Pa.

[7a] Received by the Secretary May 24, 1943.

stream. Consider also the case of A, another small town. Agitation for sewerage came from a part of A built on low ground along the banks of a stream. Just outside the town limits of A, and also on low ground, was a thickly populated area, not incorporated as a town, a part of the adjoining township. Town A provided its citizens with sewerage and sewage treatment. It was willing to take the sewage from the adjoining area and treat it, but could do so legally only by agreement with the adjoining township. The township refused to enter into any agreement for the benefit of this small area, since the remainder of the township was sparsely settled, and it was afraid of setting up a precedent.

It is noted that even the Illinois Enabling Act permits territories the freedom of choice as to whether they shall annex or disannex themselves to or from a drainage district. No thought is given to the effect of this freedom of choice on the remainder of the sanitary district. Where such a territory lies in the center of a natural drainage area, the effect of its withdrawal might be disastrous on the entire remaining sanitary district.

It is true that the reason for many of the difficulties now being encountered lies in the fact that sanitary engineering, in terms of civilization, is relatively new. The education, for great masses of people, as to its necessity has scarcely begun. It has been necessary, practicable, and advisable to construct sewerage facilities as opportunity offered—almost, it might be said, as an educational guide.

It is probably true that the educational phase is not yet over even in the United States; but, coincidental with the decline of that phase, there should be inaugurated a new phase of broad-scale planning in the organization and financing of sewerage and sewage treatment plant construction. If it is begun early it will be well done in the end; if it is begun late it may never be done at all. As a basis for such planning, the writer offers the following suggestions for an ideal design, with the thought that existing conditions and practical considerations be permitted to modify it only by the purest compulsion.

Organization.—

1. What is (or should be) the definition of a sanitary district?

A sanitary district may be defined as a section of the earth's surface consisting of the whole or part of a natural drainage area or a series of such areas draining into a common basin.

2. How extensive should a sanitary district be?

A sanitary district should be large enough to warrant the construction of a sewage treatment plant with a capacity of at least 1,000,000 gal per day when the present density of the population in the area is 20% or more of the probable ultimate density. This arbitrary definition will be further modified by certain other factors such as:

(*a*) The availability of a plant site of sufficient size for present construction and future extension.

(*b*) The average slope of the basin. Where the average slope is relatively flat the district may be limited by the time required for sewage to reach the plant in a sufficiently fresh state for proper treatment.

(*c*) Undulating or special terrains, such as a community in a flat desert area; communities built along a shore and isolated by salt marshes from the mainland, etc.

3. To what extent should existing local governmental boundaries be used in defining a sanitary district?

Pre-existing governmental boundaries should be used only where they happen to coincide with the boundaries of a drainage area. It is a purpose of this procedure to ignore all local governmental boundaries, and to consider the question of sewage entirely from an engineering point of view. It is believed that such a procedure will reduce operational costs as well as overlapping and duplicating efforts.

4. When should such a sanitary district be organized?

When the density of population of any part of the district warrants the collection and treatment of sewage. Further definitions might be set up covering this phase, but it is likely that the pressure of local opinion may be more potent.

5. Who should determine the area of a sanitary district?

It should be first approximately defined by that agency of the state government customarily charged with reviewing and passing upon proposed sewerage facilities. After the district has been organized, by the procedure outlined herein, preliminary surveys should define the area more exactly. The preliminary definition of the district is delegated to the state as being a disinterested party. The state is also considered as a centralizing agency having knowledge of the proposed organization of all districts in the same drainage area, as well as in adjoining drainage areas.

6. When should a state agency delineate a sanitary district?

On a petition from 100 qualified voters in a continuous area. It is assumed that a town or community would be the continuous area referred to.

7. How should a sanitary district be activated?

After a petition of 100 qualified voters has been referred to the state agency controlling sewage, the agency would define the approximate limits of the district. This information would then be referred to the executive branch of the state government, which would appoint three commissioners to supervise an election in the district. The state would be empowered to advance the costs of this election, from funds set up for this purpose (see subsequent heading, "Financing," regarding preliminary financing). The question to be determined by this election, of course, is that of whether the district shall be organized and sewage facilities constructed.

It is possible that, under certain conditions, it might be advisable to include in this election the selection of permanent members of the sanitary district board or commission. In other cases, assuming that the vote of the electorate is affirmative, it may be preferred to select the district board by a separate election or by appointment by the executive arm of the state. However, this step of the procedure, being entirely politico-legal, will vary from state to state.

The next step, after establishing a sanitary district board or commission, would be to proceed with a comprehensive survey of the sewage handling requirements of the district. On completion, this would be presented to the state agency for review and approval. It is assumed that the area might vary from that originally proposed by the state, but it is believed that it will seldom vary to such an extent as to affect, materially, the population included and thus require a new election.

When the state agency and the district's engineers have agreed on a comprehensive plan of the facilities, area, etc., the district is then in a position to consider costs and methods of financing.

8. What jurisdiction should a sanitary district board have?

A sanitary district board should have full jurisdiction over the collection and treatment of all sanitary wastes within its area. This should include existing facilities as well as those proposed. Storm drainage, except where it is incidental to sanitary sewerage facilities, should remain a function of existing governmental bodies. The acquisition and operation of existing sanitary facilities will constitute a special problem in every case. Additional study, in general terms, should be given to this phase.

The treatment (and transportation) of prime industrial wastes should be limited to a percentage of the total volume of sewage to be treated. That percentage should be the quantity which a plant designed for sanitary sewage can absorb without impairing its operation. Where prime industrial wastes are treated at the source they become secondary industrial wastes. A larger percentage of secondary industrial wastes might be permitted.

In general, then, the jurisdiction of a sanitary district board should be complete over all sanitary sewerage and treatment or anything which might limit or interfere with, or affect, the cost of such treatment.

9. What powers should a sanitary district have, when fully constituted?

Its powers should include the following:

(*a*) To fix the extent of present and future requirements of sanitary sewage facilities for the area, and to engage the necessary professional guidance for so fixing.
(*b*) To own, construct, or acquire such facilities as may be required.
(*c*) To operate facilities so acquired.
(*d*) To finance the construction or acquisition of such facilities by the acceptance of grants, the issuance of bonds, or such other means as can be made available.
(*e*) To make such charges for the collection and treatment of sanitary sewage as shall be found necessary to provide for adequate operation and for the retirement of indebtedness.

10. How should these powers be vested in the sanitary district?

By appropriate state legislation.

Financing.—

11. How should the early stages in the establishment of a sanitary district be financed?

The legislative act providing for the creation of sanitary districts should provide for an appropriation from state revenues. This appropriation should be used as a revolving fund for the purpose of advancing funds for elections, comprehensive surveys, etc. (but not including the acquisition of facilities), to bridge the period until permanent financing has been obtained by the district. From its financing, the district would then repay to the state moneys so advanced. In cases where the district failed to complete its organization, the state would carry the funds so advanced as a charge against any such future organization.

12. How should the construction and acquisition of sewage treatment plants be financed?

The cost of the construction and acquisition of sewage treatment plants, pumping stations, main trunk connecting sewers, outfall sewers, etc., is essentially a function of area—first, because the system is an asset not merely to the part of the district being immediately sewered, but also to those parts having no immediate need for individual sewerage; second, because modern design, no matter how limited the proposed present construction, provides for considerable extension of use before additional facilities must be constructed. There is, therefore, no equitable basis for distributing these costs except on the basis of area.

Financing by taxation on assessed value is not equitable because the value of property does not necessarily reflect its requirements for sewage facilities. Assessments based on present usage requirements are likely to work to the disadvantage of those for whom facilities are provided at a later date, or vice versa.

The owner of a hundred acres presumably can use present facilities without new construction if he should develop the property. Therefore, it follows that the costs of sewage treatment plants, etc., should be assessed on an acreage basis for all area within the district. If later development within the district requires additional construction of this type, the additional cost should again be assessed on area.

13. How should the cost of branch sewers (that is, collection systems) be financed?

The cost of branch sewers is essentially a function of accessibility or possible use. Therefore, it is a function of frontage. It follows that the cost of branch sewers should be assessed on the frontage served. Assessments should be based on the cost of an average 8-in. or 10-in. sewer at average depth. Where trunk sewers are available for individual connection, only the cost of the standard branch sewer should be assessed against frontage, the remainder of the cost being carried under paragraph 12. Costs of connections to the line of the property should be assessed against frontage on the same standard basis, such as 1 per 50 ft of frontage or, for singly-owned properties, a fraction thereof.

14. How should the cost of operating and maintaining the sewerage and treatment facilities be financed?

The cost of operating and maintaining a sanitary sewage system is essentially a function of use. Therefore, its cost should be charged against those who use it. Data presented in the Third Progress Report of the Committee

of the Sanitary Engineering Division on Sewer Rental Laws and Procedure[8] indicate that use rates are of three types: (*a*) A flat rate per connection, (*b*) a rate based on water consumption, and (*c*) a rate based on fixtures or type of connection. A flat rate is necessarily limited in application to relatively small communities with a uniform density of population. It is therefore unsuited for use in the proposed sanitary district.

Rates based on water consumption presuppose the accessibility of water records to the sewer authority. In the proposed sanitary district, this might frequently not be the case. Several different water suppliers, in some cases privately-owned water companies, might contribute to the sewage flow. As a standard method of rate fixing, therefore, water consumption is not universal enough in the possibility of its application. There is the additional factor that, with water consumption as a basis, the actual use and the presumptive use may vary widely. Extensive and unnecessary collection and treatment facilities may have to be provided which will not be used, due to the knowledge that, so long as they are not used, in terms of water, there will be no charge for these stand-by facilities.

It appears that the most equitable basis for sewage rates is that which examines and adheres the most closely to the probable use. The number and the type of fixtures, conditioned by their location as to residential or industrial use, appear to distribute the cost of operation and maintenance most satisfactorily and are the most available for universal application.

15. How should the collection of charges for sewage service be organized in a sanitary district?

A sanitary district is conceived, herein, as an administrative organization. To avoid, as fully as may be possible, the duplication of existing functions already organized, it is proposed not to authorize the sanitary district board to collect directly from the individual sewage contributor. The sanitary district would establish the assessments and rental charges, but the actual billing and collection would be in the hands of the several local governments or portions thereof that might be part of the sanitary district. They, in turn, would make periodic returns to the district. This would permit the inclusion of assessments on tax bills and, at least in certain cases, rental charges on water or other service bills issued periodically. A nominal fee for collection could then be deducted from returns to the district by the local governmental body.

Summary.—It has been the intention, in the foregoing, to propose a broad-scale plan for the organizing and financing of sewage works programs. Mr. Greeley's paper has indicated the extent to which similar planning has progressed in Illinois and some of the difficulties encountered there. From Mr. Greeley's paper, it is evident that even the Illinois Enabling Act is a compromise of political and legal expediency and that the careful planning of the engineer has been only an afterthought. Any enabling act, of course, is better than none at all, but it is the writer's thought that the engineering profession should be prepared to propose its own enabling acts in the future.

[8] *Proceedings*, Am. Soc. C. E., November, 1942, p. 1585.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

CONFORMITY BETWEEN MODEL AND PROTOTYPE A SYMPOSIUM

Discussion

BY MARTIN E. NELSON

MARTIN E. NELSON,[35] M. AM. SOC. C. E.[35a]—The conclusions reached by Mr. Soucek, in his study of the effect of submergence on the flow of water over a spillway and in his comparison of discharge over a model crest and its prototype, are verified substantially by similar tests conducted under the supervision of the writer in 1932. The model tests in question were made in connection with the design of the spillway described in the paper by Mr.

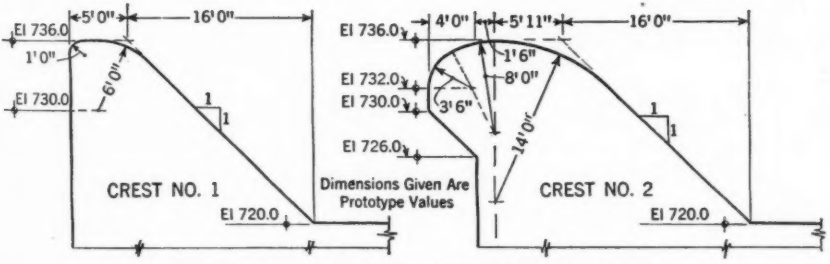

FIG. 82.—HYDRAULIC MODEL TESTS, 1 : 20 SCALE MODEL

Hopkins. The tests were made primarily for the purpose of developing a stilling basin to protect the downstream toe of the dam. Two model spillway crests with cross sections as shown in Fig. 82 were constructed of concrete to dimensions $\frac{1}{20}$ as large as the proposed prototype. Model crest No. 1, simu-

NOTE.—This Symposium was published in October, 1942, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: December, 1942, by A. E. Niederhoff, Assoc. M. Am. Soc. C. E.; January, 1943, by Messrs. C. I. Grimm, and Joe W. Johnson; February, 1943, by V. L. Streeter, Assoc. M. Am. Soc. C. E.; March, 1943, by Glen N. Cox, M. Am. Soc. C. E.; April, 1943, by Messrs. Graham Walton, H. A. Einstein, K. G. Tower, R. J. Pafford, Jr., Edward H. Schulz, and F. T. Mavis; May, 1943, by Messrs. D. C. McConaughy, L. Standish Hall, A. J. Gilardi, Fred W. Blaisdell, A. R. Thomas; and June, 1943, by Messrs. Gilbert H. Dunstan, Robert F. Kreiss, J. H. Douma, and Marvin J. Webster.

[35] Senior Engr., U. S. Engr. Office, St. Paul, Minn.

[35a] Received by the Secretary March 22, 1943.

lating the original proposed design, was equipped with several pressure piezometers along its center line. On this model it was observed that, under certain conditions of discharge and head, considerable negative pressure formed under the nappe. A model of crest No. 2 was tested to determine the effect of the indicated modification in shape on the pressure gradient under the nappe.

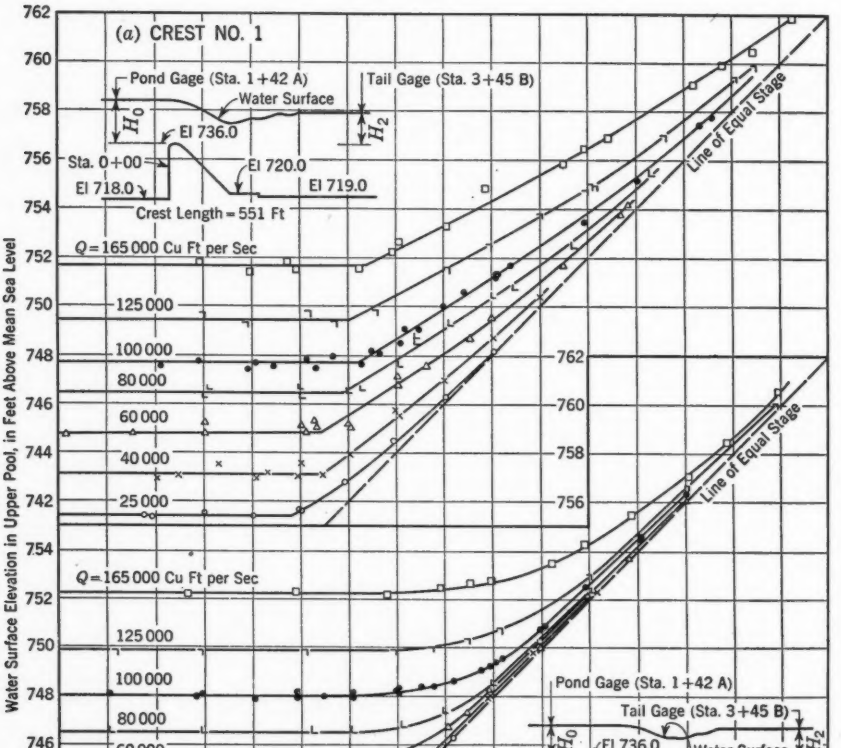
(a) CREST NO. 1
Pond Gage (Sta. 1+42 A)
Water Surface
Tail Gage (Sta. 3+45 B)
H_0
H_2
El 736.0
Sta. 0+00
El 720.0
El 718.0
El 719.0
Crest Length = 551 Ft
Q = 165 000 Cu Ft per Sec
125 000
100 000
80 000
60 000
40 000
25 000
Line of Equal Stage
(b) CREST NO. 2
Water Surface Elevation in Upper Pool, in Feet Above Mean Sea Level
Water Surface Elevation in Lower Pool, in Feet Above Mean Sea Level
730 732 734 736 738 740 742 744 746 748 750 752 754 756 758 760 762
740 742 744 746 748 750 752 754 756 758 760 762

FIG. 83.—STAGE RELATION CURVES

Although the pressure condition was considerably improved, for economic reasons and because the condition under which the negative pressures in the prototype might be serious would be of short duration, this design was not adopted for the prototype. However, the calibrations of model crest No. 2 will be introduced in this discussion to indicate the change in discharge coefficients for free and submerged conditions with respect to crest profile. The

prototype spillway was constructed to the section designated as crest No. 1 in Fig. 82, except that the crest elevation is 737.5 instead of 736.0. The spillway as constructed is 551 ft long.

The models, 2.53 ft long, representing spillway lengths of 50.6 ft, were tested in a glass-walled flume. The floor of the upstream approach was fixed at El. 718.0 and that of the downstream approach at El. 719.0. The water

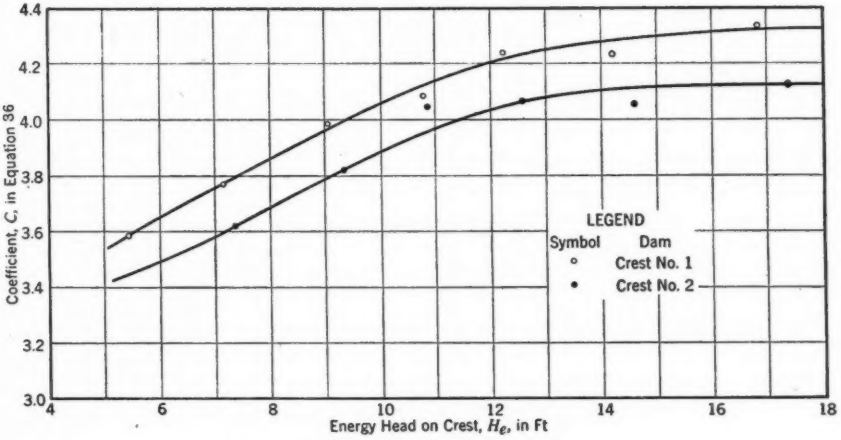

FIG. 84.—COEFFICIENTS FOR FREE DISCHARGE

supply was measured over a V-notch weir, and tailwater levels were regulated by means of an adjustable gate at the lower end of the testing flume. Water-surface elevations in the upper pool were observed by means of a piezometer in the flume wall, together with a stilling well and hook gage, at a point corresponding to 142 ft upstream from the crest, and similar readings in the lower pool were made at a distance of 345 ft downstream from the crest. In a typical test the water supply in the model was adjusted to provide a discharge corresponding to $\frac{50.6}{551}$ times an arbitrarily selected quantity of river discharge. When the flow had become steady and the tailwater level had been adjusted to a desired elevation, several readings were made on the weir gage and on the upper and lower pool gages. While the water supply was maintained at the same rate, another water level was simulated in the lower pool, and the gages were read again. This procedure was followed until a sufficient number of points had been obtained to describe the headwater-tailwater relation for the given discharge. Then, a different rate of flow was established and the tests were repeated. Seven discharge rates ranging from 25,000 to 165,000 cu ft per sec were simulated.

The headwater-tailwater relations observed on the models of crest No. 1 and crest No. 2 are shown in Fig. 83. The coefficient C in the formula (see Eq. 13),

$$Q_p = C\,L\,H^{1.5}_e \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (36)$$

computed for each discharge rate in the region of free flow is shown graphically with respect to total head in Fig. 84. Coefficients of discharge under submerged flow conditions were also computed by Eq. 13 (or Eq. 36) for each crest at several stages of submergence. Fig. 85 contains the ratios of coeffi-

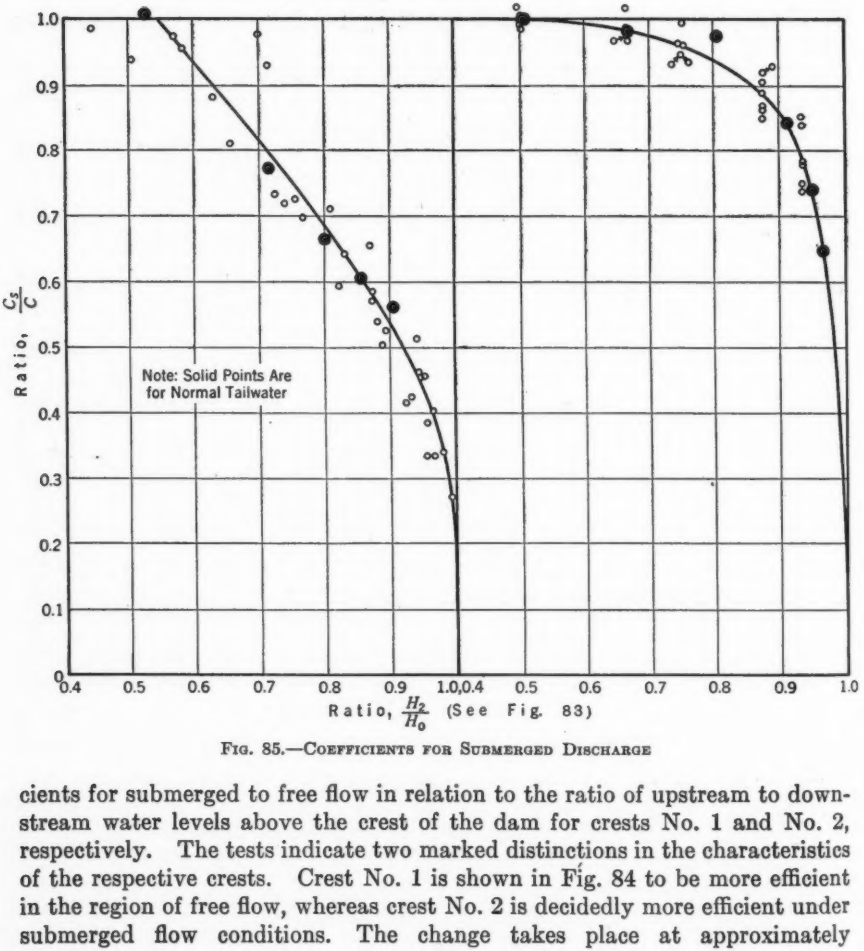

FIG. 85.—COEFFICIENTS FOR SUBMERGED DISCHARGE

cients for submerged to free flow in relation to the ratio of upstream to downstream water levels above the crest of the dam for crests No. 1 and No. 2, respectively. The tests indicate two marked distinctions in the characteristics of the respective crests. Crest No. 1 is shown in Fig. 84 to be more efficient in the region of free flow, whereas crest No. 2 is decidedly more efficient under submerged flow conditions. The change takes place at approximately $\frac{H_2}{H_0} = 0.50$, the point below which the submergence has no apparent effect on the coefficient ratio. These results verify conclusion (*a*) reached by Mr. Soucek: "The effect of submergence on the discharge is affected by the shape of the spillway * * *." However, contrary to Mr. Soucek's findings, the tests on crest No. 1 and crest No. 2 showed no consistent relation between the rate of discharge and the coefficient pattern.

Comparison of Model and Prototype.—After the prototype had been constructed, the water-surface elevations in the upper and lower pools were

observed on staff gages at frequent intervals during the passage of several peak rates of discharge over the crest. From these readings, hydrographs were constructed from which the simultaneous elevations of the upper and lower pools were read at the peak of the hydrograph when the flow could be assumed to be approximately constant. The relation of the water levels observed during several freshets is shown in Fig. 86. In the same figure is

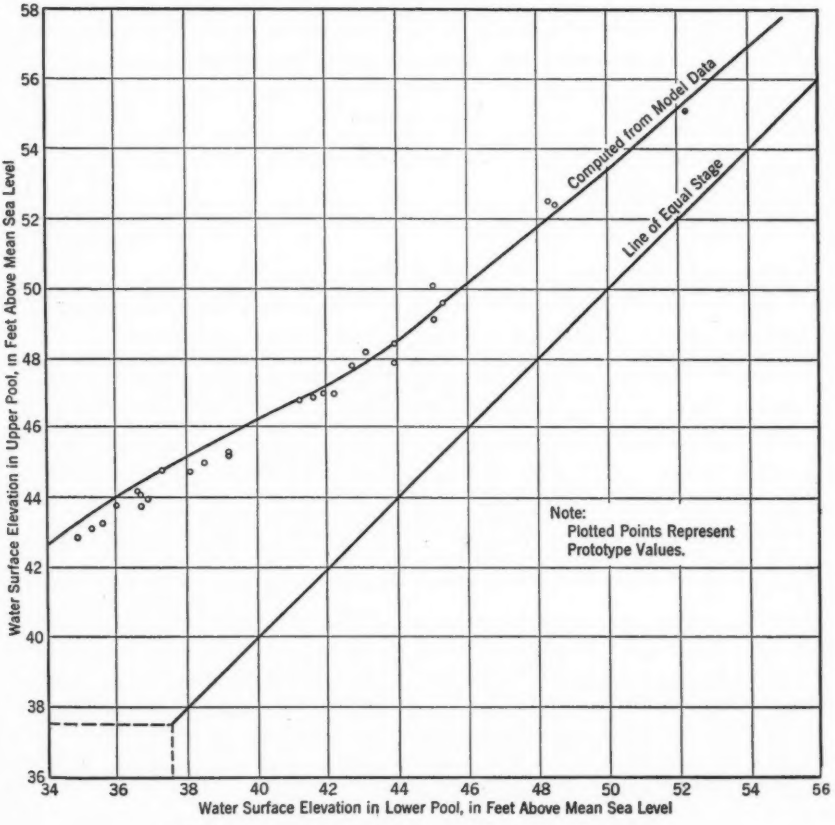

FIG. 86.—STAGE RELATION CURVES—MODEL AND PROTOTYPE

shown this relation computed from the discharge coefficients determined in the model. The pool relation curves shown in Fig. 83(*a*) could not be compared directly with the data observed in the prototype because the prototype crest was constructed to a higher elevation than the model crest and, furthermore, because the tailwater rating curve existing in the prototype was different from that used in the model. This correlation between the model and prototype is considered to be very satisfactory and substantiates Mr. Soucek's conclusion (*d*)—"Reasonably good agreement may be expected between spillway models and prototypes of the sizes used in these tests under submerged as well as free flow conditions."

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

STRAINS, STRESSES, AND SHEAR IN ENGINEERING PROBLEMS

Discussion

BY MESSRS. C. A. P. TURNER, AND MARK W. HUGGINS

C. A. P. TURNER,[58] M. AM. SOC. C. E.[58a]—Illogical deductions (see heading, "Shear and Stress") presented in Mr. Woodard's paper are that:

> "* * * (*a*) in materials strained by exterior forces including shearing forces, only two kinds of stresses are set up—compressive stresses and tensile stresses—and (*b*) shear is accompanied by and resisted by internal forces of tension and compression. In other words, there is no such thing as shearing stress or shearing strength as distinguished from compressive and tensile stresses and compressive and tensile strength."

Axial forces cause elongation: Positive if in tension, negative if in compression. The unit stress under these conditions is $s = \frac{P}{A}$. In simple elongation, rotational strains accompany elongation. These are best visualized by a strip of rubber 1 in. in width marked off in 1-in. squares. By drawing diagonal lines from corner to corner of the squares with white lead, a kind of pantagraph frame is formed by the white lines. If the strip is stretched, these diagonal lines rotate about their intersection, disclosing the angular distortion of two-dimensional displacement. These rotational actions differentiate rotational shear strain from simple elongation.

Pure shear is found in the torsion of the shaft or the twist in a square flat plate under load supported on four sides, or in the case of continuous plate supported by spaced columns. Fig. 12 represents a stress-line analysis of the latter type of plate, and the traces of the spiral right and left shears on top and bottom surfaces are presented in the diagram. The coordinates of these shears are the planes of zero shear and greatest vertical shear and of zero shear and greatest twist. These shears fix the contours of curvature under load by

NOTE.—This paper by Silas H. Woodard, M. Am. Soc. C. E., was published in February, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1943, by Messrs. C. M. Goodrich, Maurice P. van Buren, Joseph A. Wise, and David B. Hall; and June, 1943, by Messrs. Roberto Contini, I. Nelidov, Bernard L. Weiner, Richard W. Albrecht, and Duff A. Abrams.

[58] Cons. Engr., Columbus, Ohio.

[58a] Received by the Secretary June 1, 1943.

their intersections. The contours are principal stresses at the middepth of the plate. The twisting shears which develop the contours make variable angles with the principal strain far different from 45° as given by the author. Warping of the plate generates twisting couples about the axis of x, y, and z. The sign of the twisting couples changes at the point of contraflexure and at the principal planes.[59]

Shears and Principal Stresses in Beam Action.—When a load is placed on a simply supported beam, the beam bends down under the load. The top fibers are shortened and the bottom fibers are stretched, causing a trapezoidal distortion on the vertical face of the beam. The reaction of these positive

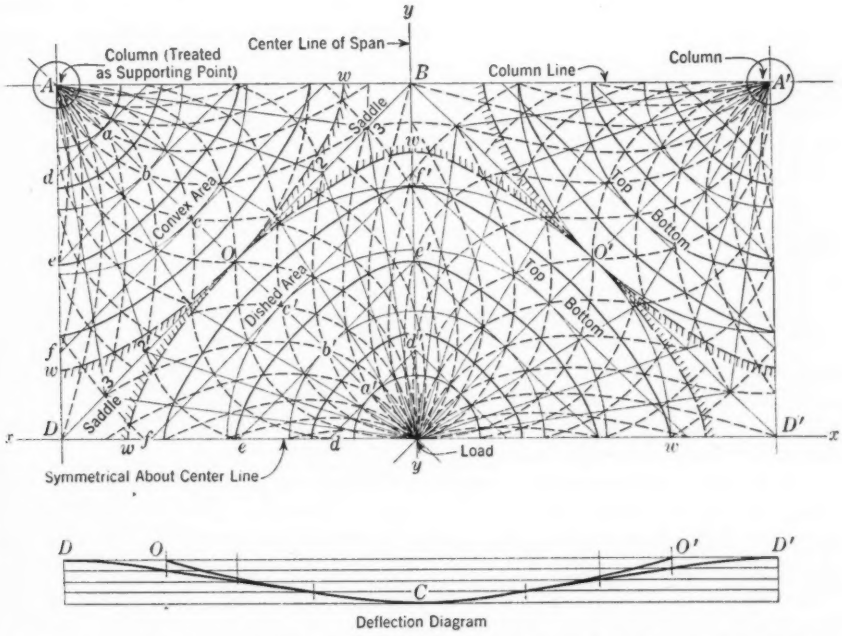

FIG. 12.—STRESS-LINE ANALYSIS OF A COLUMN SUPPORTED, CENTRALLY LOADED, FLAT SLAB

and negative deformations causes shearing stresses in the material as illustrated by the deportment of two planks of the same length, one on top of the other, with the ends in vertical planes. As the planks bend under the load, the lower corners of the upper plank slide by the upper corners of the lower plank; and, when arranged in this way, the combined stiffness and load capacity is that of two planks. If, instead of allowing the planks to slide one upon the other, they are glued together, the strength is then equal to that of four single planks and the stiffness is increased to that of eight single planks, sliding the one on the other.

[59] "Elasticity, Structure and Strength of Materials Used in Engineering Construction," by C. A. P. Turner, Pt. 1, pp. 286–292.

The stress brought on the glue is a shearing stress, and its relative intensity at any point is proportional to the amount of sliding that would occur were the planks not so glued together. This shear causes rhombic distortion, which, in a simply-supported beam, increases from zero at the center to a maximum at the end and, in a restrained beam, increases from the center to the point of inflection, decreasing from that point to zero at the support.

The vertical cross section at midspan is the critical section resisting horizontal elongation of bending, whereas the horizontal longitudinal cross section at middepth resists the horizontal shearing stress. Because the latter cross section is far greater in area than the former, the deflection from the shear strain to which it is subjected is so small in a rectangular section when the length is greater than twelve times the depth that it is neglected in the common or approximate theory of flexure.

The building codes of all North American cities recognize three kinds of beam failure: (1) Compression failure caused by crushing the top fibers under maximum moment; (2) tension failure caused by breaking the bottom fibers in tension; and (3) shear failure caused by combination of rhombic and trapezoidal shear strains. The latter are measured by the sharpness of curvature.

For safety, building codes provide limiting fiber stresses for tension and compression, and safeguard against shear failure by specifying fixed limits for the value of the quantity, external shear force, divided by the area of the beam at the support.

Thus, by observation, building code authors recognize shear failure in beam resistance and are a step in advance of Mr. Woodard's thesis. However, the safeguard which the building codes provide against shear failure is founded on complacent ignorance as to the locus[60] of greatest strain; also there is wide[60] divergence as to its magnitude under the same external shear, depending on the distribution of load and the restraint at the support.

In treating nonhomogenous concrete by the same theory as homogenous material, Mr. Woodard falls into an interesting error. The neutral plane of homogenous and concrete beams is the locus of consecutive states of right shearing stress, functioning as centers of rotation for each elemental section. Incidentally, in the homogenous beam, this locus is where tension and compression balance, and principal tension and compression make angles of 45° with the plane. Otherwise, the locus is in the reinforced concrete beam,[61] because the tension of the steel at the bottom equilibrates the compression in the concrete by bond shear at the surface of the steel, instead of at the neutral plane. As a result, the states of right shearing stress as centers of rotation in the neutral plane are twisted 22.5° [61] from their position in the homogenous beam. Consequently, the principal compression and horizontal shear intersect the neutral plane at 22.5°, and the curve of vertical shear and principal tension intersects that plane at 67.5°.[61] Therefore, elastic deflection of the concrete beam increases more rapidly than the load, because the neutral plane is compressed under load.

[60] "Elasticity, Structure and Strength of Materials Used in Engineering Construction," by C. A. P. Turner, Pt. 1, pp. 166–167, 192–194.

[61] *Ibid.*, pp. 234–235.

Mr. Woodard's treatment of torsion of a shaft is erroneous. The torsional moment is coordinated readily with the shear stress on a transverse section without reference to tension or compression in the material. The simplest solution is that of shear stress volume times lever,[62] which agrees with Rankine's analysis.[63]

The distribution[64] of tensile and compressive stresses and their reverse summation precludes the possibility of coordinating the torsional moment with their reactions. On a round shaft a neutral cylinder[64] (radius, 0.7071 r) is developed, on which every element is under no strain; but inside this cylinder every fiber parallel to the axis is stretched, and outside it every fiber is compressed. In the cross section all radial lines are under compression, increasing from the outer fiber to the axis; and all circular lines are under tension, which is at a maximum at the outer fiber and decreases toward the axis. Pure shear on the cross-sectional area has the same unit intensity from the axis to the surface. The spiral tensions and compressions on the surface of the shaft and each annular cylinder within intersect the cross section of pure shear at 45° only at the neutral cylinder. Their obliquity changes from 45° as radically as the twisting shears in the plate problem of Fig. 12.

The complexity of the three-dimensional strain of torsion precludes coordination of the reaction of tensile and compressive stresses with the torsional moment.

MARK W. HUGGINS,[65] ESQ.[65a]—Although the writer cannot agree with the suggestions presented in this paper, he feels that its publication is certain to fulfil a useful purpose, if only by stirring up further discussion of this most important problem. The importance of this subject does not appear to be realized fully by engineers in general. The truth of this statement will be apparent to any one who studies any of the modern specifications for allowable stress. So far as the writer is aware, no specification attempts to outline any procedure which may be followed in determining safe stresses in cases of combined stress. The same stress, which is considered safe in the case of pure tension also may be used in designs in which a complex state of stress exists, provided only that the maximum principal stress does not exceed the specified limiting value for pure tension. Numerous tests, such as those published in Table 1 of the paper and in many texts on strength of materials[66] illustrate the inaccuracy of this procedure.

Any suggestion for changing the standard nomenclature of mechanics of materials is certain to lead to nothing but confusion. It has been accepted generally that the term "stress" shall apply to true forces and that "strain" shall apply to true deformation. The use of the term "stress" to define some artificial quantity, which is always proportional to strain, is not advisable as

[62] "Elasticity, Structure and Strength of Materials Used in Engineering Construction," by C. A. P. Turner, Pt. 1, pp. 11–12.

[63] "Applied Mechanics," by W. J. M. Rankine, Arts. 320–321, pp. 353–355.

[64] "Elasticity, Structure and Strength of Materials Used in Engineering Construction," by C. A. P. Turner, Pt. 1, pp. 239–251.

[65] Asst. Prof. of Civ. Eng., Univ. of Toronto, Toronto, Canada.

[65a] Received by the Secretary July 6, 1943.

[66] "Strength of Materials," by John Case, Edward Arnold & Co., London, 1938, 3d Ed., pp. 85–96.

the same result might be attained just as well by setting an upper allowable limit on strain rather than on stress.

The author states that there is no need to consider "an imaginary and inexplicable thing called shear stress" in obtaining an answer to the question of how shear is resisted. He maintains that stresses are limited to tensile and compressive stresses. This hypothesis does not give a very satisfactory interpretation of the cup-cone type of failure which occurs in tension test pieces of mild steel, in which the effect of the ends would not appear to have any considerable effect on the type of failure, provided that the length of the reduced-area section is sufficiently large. The final failure is obviously of a shear type and occurs across a plane for which the normal strain is less than that in the axial direction. The initial cause of failure, in this case, is sliding along 45° planes, which produces necking. At a certain stage in the loading, the cohesive resistance of the middle part of the bar is overcome, causing a crack of the brittle type, but, at the same time, yielding by sliding continues near the surface and the final sliding fracture occurs near the boundary.[67]

[67] "Strength of Materials," by S. Timoshenko, Pt. II, 2d Ed., 1941, pp. 418–421.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

A SYMMETRICALLY LOADED BASE SLAB ON AN ELASTIC FOUNDATION

Discussion

BY MESSRS. ROBERT O. JAMESON, AND L. J. MENSCH

ROBERT O. JAMESON,[5] M. AM. SOC. C. E.[5a]—Although the subject of beams on elastic foundations has been treated at some length by other authors,[6,7] Mr. Benscoter has simplified the solution of this special case greatly by use of moment distribution. The formulas and graphs presented should prove valuable timesavers.

The complete formulas for slope, moment, and shear (compare Eqs. 24) are:

$$\theta(x) = \frac{dz}{dx} = \frac{F\lambda^2}{2EI}\left[(v-u)\sin\frac{x}{\lambda}\cosh\frac{x}{\lambda} + (u+v)\cos\frac{x}{\lambda}\sinh\frac{x}{\lambda}\right] + \theta_a\left[(v+u)\sin\frac{x}{\lambda}\cosh\frac{x}{\lambda} + (u-v)\cos\frac{x}{\lambda}\sinh\frac{x}{\lambda}\right] \quad \ldots\ldots (36a)$$

$$M(x) = EI\frac{d^2z}{dx^2} = -F\lambda\left(u\sin\frac{x}{\lambda}\sinh\frac{x}{\lambda} - v\cos\frac{x}{\lambda}\cosh\frac{x}{\lambda}\right) + \frac{2EI\theta_a}{\lambda}\left(v\sin\frac{x}{\lambda}\sinh\frac{x}{\lambda} + u\cos\frac{x}{\lambda}\cosh\frac{x}{\lambda}\right) \quad \ldots\ldots (36b)$$

and

$$V(x) = EI\frac{d^3z}{dx^3} = -F\left[(u+v)\sin\frac{x}{\lambda}\cosh\frac{x}{\lambda} + (u-v)\cos\frac{x}{\lambda}\sinh\frac{x}{\lambda}\right] + \frac{2EI\theta_a}{\lambda^2}\left[(v-u)\sin\frac{x}{\lambda}\cosh\frac{x}{\lambda} + (u+v)\cos\frac{x}{\lambda}\sinh\frac{x}{\lambda}\right] \quad \ldots\ldots (36c)$$

Some designers prefer to plot two moment curves for a restrained beam: One representing positive moments due to vertical loads on the simply supported beam, and the other showing negative moments due to restraint at

NOTE.—This paper by Stanley U. Benscoter, Jun. Am. Soc. C. E., was published in May, 1943, *Proceedings*.

[5] Care, U. S. Engrs., Southwestern Div., Dallas, Tex.

[5a] Received by the Secretary July 6, 1943.

[6] "Strength of Materials," by S. Timoshenko, Pt. II, 2d Ed., 1941, pp. 1–24.

[7] *Engineering News-Record*, Vol. 93, p. 881 (letter to the editor by C. W. Pantke).

ends of the beam. The equations for finding these moments are as follows: Referring to Eq. 36(*b*) when $M_a = 0$,

$$\theta_a = F \frac{\phi_1(a)}{\phi_3(a)} \quad \ldots\ldots (37)$$

Substituting Eq. 37 in Eq. 36*b*,

$$M(x) = C_1 F \lambda \quad \ldots\ldots (38)$$

in which

$$C_1 = \frac{1}{\lambda}\left[\frac{\phi_1(a)\,\phi_3(x)}{\phi_3(a)} - \phi_1(x)\right] \quad \ldots\ldots (39)$$

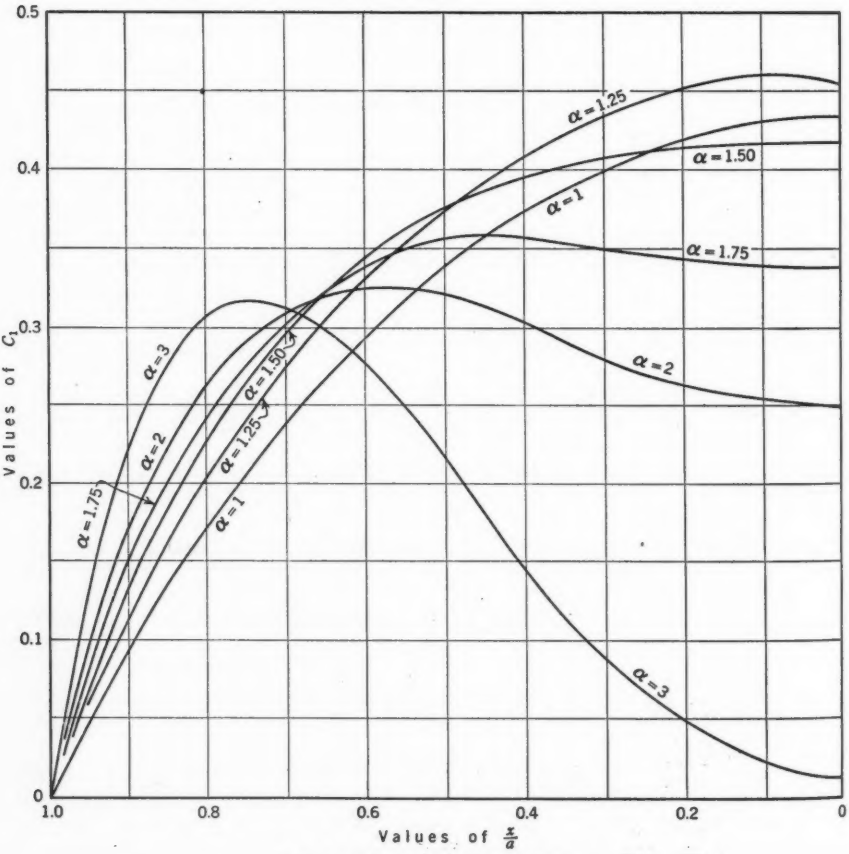

FIG. 9.—POSITIVE MOMENT FACTORS

Values of C_1 are plotted in Fig. 9. Eq. 38 is used in finding the positive moments on the beam.

With no external loads on the beam, $F = 0$ in Eq. 36*b*, and

$$\theta_a = \frac{M_a}{\phi_3(a)} \quad \ldots\ldots (40)$$

Substituting Eq. 40 in Eq. 36*b* gives

$$M(x) = D(x)\, M_a \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

in which

$$D(x) = \frac{\phi_2(x)}{\phi_3(a)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

Values of $D(x)$ are shown in Fig. 10. These values may be used also in solving Eq. 10*c*. The moments due to end restraint only can be determined by Eq. 41.

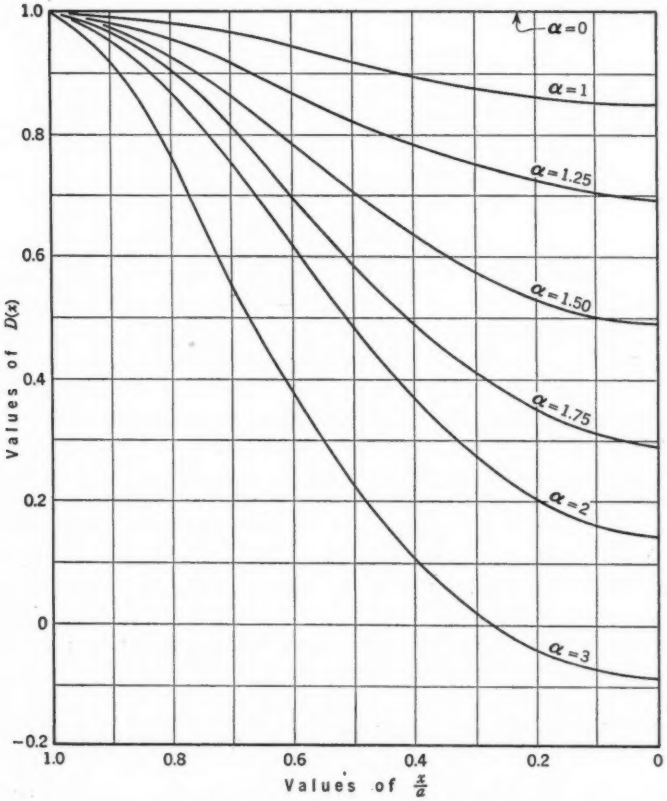

FIG. 10.—RESTRAINT MOMENT FACTORS

The example shown in Fig. 7(*a*) in the paper is solved by use of Eq. 38 and Fig. 9 as follows: $\alpha = 1.74$; $x = 0$; $\lambda = 11.5$; $C_1 = 0.343$ (from Fig. 9); and—

$$M_0 = 0.343 \times 12.4 \times 11.5 = +\,48.9$$

Again, for $x = \lambda$; and $C_1 = 0.347$:

$$M(\lambda) = 0.347 \times 12.4 \times 11.5 = +\,49.5$$

For solving the example shown in Fig. 7(*b*), Eq. 41 and Fig. 10 are used, thus: $\alpha = 1.74$; $x = 0$; $M_a = 149$; $D(x) = 0.306$; and—

$$\begin{aligned} -M_0 &= 0.306 \times 149 = -45.6 \\ +M_0 \text{ (from Fig. 7(}a\text{))} &= +48.9 \\ M_0 \text{ (net)} &= +3.3 \end{aligned}$$

Also, $\alpha = 1.74$; $x = \lambda$; $D(x) = 0.673$; and—

$$\begin{aligned} -M(\lambda) &= 0.673 \times 149 = -100.3 \\ +M(\lambda) &= +49.5 \\ M(\lambda) \text{ (net)} &= -50.8 \end{aligned}$$

For the example shown in Fig. 8, the center moment (for $\alpha = 2$; $C_1 = 0.248$; $D(x) = 0.144$; and $\lambda = 2$) is

$$\begin{aligned} +M_0 &= 0.248 \times 12 \times 2 = +5.95 \\ -M_0 &= 0.144 \times 11.6 = -1.67 \\ M_0 \text{ (net)} &= +4.28 \end{aligned}$$

L. J. MENSCH,[8] M. AM. SOC. C. E.[8a]—Many able mathematicians have worked on the problem presented in this paper, and all of them have based their analyses on the assumption that Eq. 13 holds true. Any experienced engineer who seriously reflects on the requirements in such a case or makes the simplest kind of experiment with a card or wooden board must predict that this assumption plainly is not feasible except in rare instances. It did not apply in the case of a rail supported by ties as treated by the late A. N. Talbot,[9] Past-President and Hon. M., Am. Soc. C. E., although the error was probably less than 25%. Frost action will make k unsteady and reduce its value to possibly 30 to 50 lb per cu in. The mathematics are valid enough, but they do not prove the assumption.

Eq. 12 means neither more nor less than that the intensity of the forces acting on the slab per unit length equals w. It is identical with the fundamental equation of flexure in elementary textbooks:

$$E\,I\,\frac{d^2M}{d^2x} = w \qquad (43)$$

Once the computer can make a good guess at its distribution, the problem can be solved easily by simple mathematics familiar to busy structural engineers. The writer has shown elsewhere[10,11] that the distribution of w can be assumed as shown in Fig. 11(*a*) and that the deflection of the slabs will be approximately as in Figs. 11(*b*) and 11(*c*), and will rarely be as shown in Fig. 11(*d*). In most

[8] Civ. Engr. and Contr., Chicago, Ill.

[8a] Received by the Secretary July 14, 1943.

[9] Progress Report of the Special Committee to Report on Stresses in Railroad Track, *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1207.

[10] "Joints for Concrete Pavements," by L. J. Mensch, Chicago, Ill., 1935.

[11] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1148.

cases the slab will rise from contact with the soil in the central part, which makes Eq. 13 invalid.

The design of a water channel or large tank is by no means as simple as would appear from the paper, and serious mishaps with such structures have been encountered by engineers of standing in the profession. The engineer, experienced in this type of structures, will attack the problem in the author's "Illustrative Examples," as follows:

1. He will ask himself what a heavy rainstorm of 2-in. to 6-in. precipitation in as many hours will do to the structure which is being constructed. Will the banks cave in at one or both sides of the channel at various places and trap the water so that an uplift is produced which may shove the structure sideways a few feet or lift it up entirely and break it? This phase of the project alone will involve quite some study and expense.

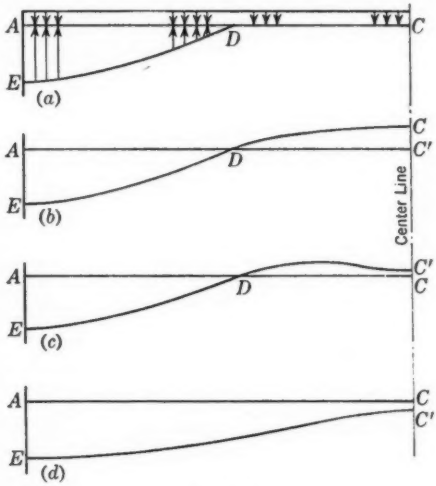

FIG. 11

2. Suppose that the engineer is in a position to guard his structure against a highwater level not exceeding 15 ft above the subgrade. Then the base slab will have to be strong enough to withstand the moment of 148 ft-kips mentioned by the author, as the water pressure will just balance the weight of the structure.

3. Let it be now assumed that conditions are favorable and no hydraulic uplift has to be taken in consideration and that the engineer, induced by the paper, will proceed to investigate the stresses in the base slab due to the wall load alone. He or his assistants will not have the time or patience to check the mathematics of the paper, especially in the absence of any bibliography on this subject, and instead likely will make a simple test by laying a thin ruler about 1 ft long with the flat side on the pages of an open book and by pressing the ends down with a force of about 20 lb. They will notice a pronounced depression near the ends and a slight tendency of lifting up at the middle. Considering that it is good practice to cantilever the base slab beyond the wall from 6 in. to several feet, they may assume as a first approximation that the base slab deflects as shown in Fig. 12.

They may assume the line ED as a common parabola; line AE equals z_a in the author's nomenclature and, following the author's idea, they will assume the soil pressure, w, proportionate to the ordinates of the parabola on the length AD. The length of the half slab AC will be loaded by the uniform dead load of 375 lb per sq ft, the weight of the wall is 12,400 lb per lin ft; and therefore one can write, for the vertical loads—$12{,}400 + 26 \times 375 = \frac{2}{3} w_a \times 16$, from which $w_a = \dfrac{22{,}150}{10.67} = 2{,}070$ lb per sq ft or 14.4 lb per sq in.

The moments acting in the slab can be found by simple statics: $M_C = 22{,}150 \times (26 \text{ ft} - \frac{3}{8} \times 16 \text{ ft}) - 12{,}400 \times 22 \text{ ft} - 375 \times \frac{26^2}{2} = 43{,}000$ ft-lb (tension at bottom). The maximum negative moment will be found at the section where the shear is zero; it is considerably smaller than the positive moment. The point C′ closely approaches the point C.

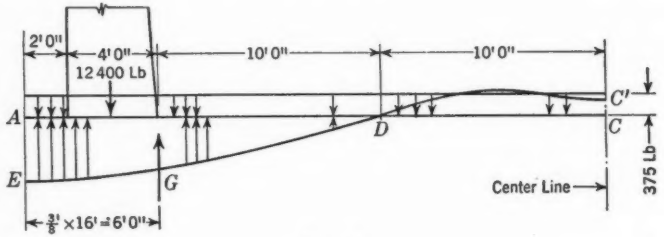

FIG. 12

No radical change will be found when the line of deflection is assumed to intersect the line AC nearer or farther from C than the 10 ft shown in Fig. 12.

4. The experienced engineer is accustomed to design the base of a water channel or tank for actual service conditions; namely, for the wall backfilled, and for the structure filled with water (in this example assumed to be to a height of 25 ft). When hydraulic uplift is not encountered, it is not economical in a structure of considerable width (sometimes several hundred feet wide) to make the base slab of uniform thickness, and, in order to avoid the use of piles in ground of medium carrying capacity, the experienced engineer most likely will assume the base slab to cantilever 4 ft beyond the wall and will make it 30 in. thick for only about 10 ft inside the structure. He will make the remaining slab only about 6 in., or perhaps 8 in., thick, and will connect it with the thicker part by a well-designed metal water stop, as shown in Fig. 13. The engineer is now confronted with a number of decisions before he can proportion the foundation slab of this structure. What assumption shall be made for the earth pressure acting on the wall? Shall he assume the customary value of 15 H^2 for the pressure per linear foot of wall, and the moment about the base as 5 H^3, or is the moment of 4 H^3 a better guess?

Some engineers, who have seen shrinkage cracks develop in backfills near the wall after heavy rains, claim that the earth pressure should not be counted upon to relieve the pressure on the wall from the water on the inside. Others go as far as to say that colloidal tension may exert a pull on the wall that will increase the effect of the water pressure. Others have observed considerable deflections in concrete walls, not tied on the top, and claim that passive earth pressure should be taken into account. When the backfill consists of good material, such as sand, gravel, stone, and slag, and is thoroughly rolled or tamped, it requires a movement of about $\frac{H}{200}$ at the top to develop a passive

pressure about four times the active pressure, whereas when the fill simply is dumped from cars or grab buckets, it will take a movement of $\frac{H}{50}$ to $\frac{H}{100}$ to develop the aforementioned passive pressure. The movements given take into account time loading. Suppose the decision has been made that the combined water and earth pressure on the wall exerts a moment of 80,000 ft-lb at the base of the wall for each linear foot.

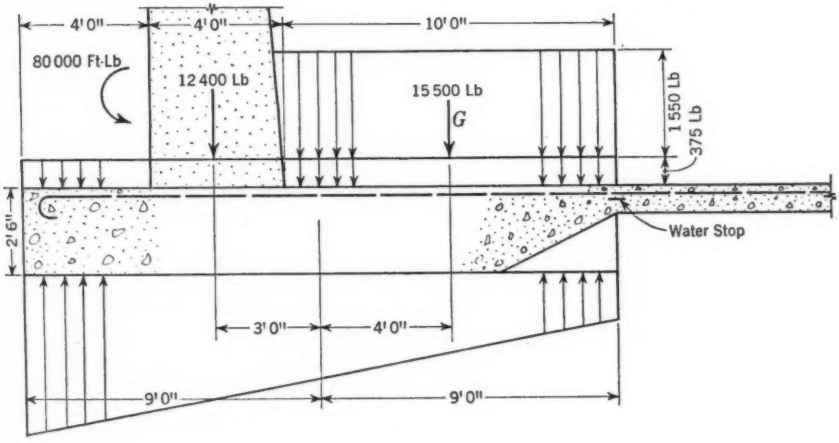

FIG. 13

The sum of all vertical loads on the foundation will be $12{,}400 + 375 \times 18 + 1{,}550 \times 10 = 34{,}650$ lb and the ideal distributed soil pressure $\frac{34{,}650}{18} = 1{,}925$ lb per sq ft. This pressure will be modified by the moment acting at the base of the foundation slab: $80{,}000 + 12{,}400 \times 3 - 15{,}500 \times 4 = 55{,}200$ ft-lb which produces an ideal plus-and-minus stress of $\frac{55{,}200}{\frac{1}{6} \times 18^2} = 1{,}022$ lb per sq ft, or the soil pressure at A = 2,947 lb per sq ft and that at D = 903 lb per sq ft. The differential pressure of 2,000 lb per sq ft will tilt the base about 0.14 in. for a soil modulus of 100 which will result in a horizontal movement of the top of the wall of about $\frac{1}{4}$ in. The elastic deformation of the wall will increase this movement to about $\frac{1}{2}$ in., or $\frac{H}{720}$, which will bring into action part of the passive pressure of the backfill.

5. When the structure is built on wet clay it is possible that the outward pressure of the water (diminished by the earth pressure) will cause a sliding of the foundations, as shown in Fig. 13. This outward pressure has been assumed at about 10,000 lb per lin ft, whereas the weight on the foundation slab is 34,650

lb; and the coefficient of friction required $\frac{10{,}000}{34{,}650} = 0.3$. Conservative engineers do not allow more than 0.2 as the coefficient of friction on wet clay. It will be necessary to run reinforcing steel from the 30-in. slab into the thin slab across the contraction joint, a practice that many engineers condemn but that has been used on some of the largest and most successful reservoirs built in the United States.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

RELATION OF UNDISTURBED SAMPLING TO LABORATORY TESTING

Discussion

BY D. M. BURMISTER

D. M. BURMISTER,[10] Assoc. M. AM. SOC. C. E.[10a]—The extent to which the strength properties of clay may be affected by disturbance during sampling is treated in this worth-while contribution to soil mechanics. The author has suggested fairly reliable methods for determining whether a sample may be considered relatively undisturbed. There are some points, in connection with the curves and data presented, on which the writer wishes to comment.

The consolidation curve CP1, for example, in Fig. 3 represents a relatively undisturbed sample, which possesses the greater part of its natural structure at the preconsolidation pressure of approximately 0.5 kg per sq cm. This natural structure, which is the source of the strength of the clay, may be that visualized by Professor Casagrande or that due to physicochemical processes, as believed by Professor Terzaghi. The maximum effect of disturbance, resulting from a complete remolding of the material at approximately constant moisture content, is shown by the vertical decrease in void ratio under constant pressure (0.5 kg per sq cm) to curve CP3. The vertical void-ratio intercept represents the change in void ratio (and hence settlement) required before equilibrium can be achieved under the same pressure, as a result of complete disturbance. The remolded material now possesses an entirely different type of structure, which involves a complete breakdown in the natural bonds between grains. The structure is characterized by a homogeneous, dispersed state of the "remolded structure." The void ratio at any given pressure, which is consistent with this structure at the equilibrium condition, is much smaller than for the undisturbed material. Therefore, having reached equilibrium under this pressure at a smaller void ratio, the material will compress less in

NOTE.—This paper by P. C. Rutledge, Assoc. M. Am. Soc. C. E., was published in November, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1942, by Messrs. Benjamin K. Hough, Jr., and F. M. Van Auken; March, 1943, by Jacob Feld, M. Am. Soc. C. E.; April, 1943, by Messrs. Raymond F. Dawson, and Hamilton Gray; and June, 1943, by Messrs. Karl Terzaghi, and D. P. Krynine.

[10] Asst. Prof., Civ. Eng., Columbia Univ., New York, N. Y.

[10a] Received by the Secretary August 2, 1943.

the next load increment than does the undisturbed material. This means that the consolidation curves for the undisturbed and remolded clays must always intersect at some pressure (40 to 50 kg per sq cm in this case), as shown in the upper part of Fig. 4, where the curves for this material are reproduced. The magnitude of this pressure depends primarily on the mineralogical character of the clay.

The vertical intercept between consolidation curves for the undisturbed and remolded clay at the preconsolidation pressure is one important physical measure of the natural structure the clay possesses, and indicates the maximum effect of complete disturbance. It is to be noted that, at higher pressures, structure becomes less important and the effect of disturbance is less. This indicates that structure is more important for clays consolidated under relatively light loads, and that the effect of disturbance during sampling on the consolidation properties is likely to be greater than for clays consolidated under heavy loads.

It is believed that the consolidation curves intersect, not because the undisturbed clay becomes completely remolded by compression, as suggested by the author, but because a stage is reached at which a characteristic change in the compression phenomena occurs for the given clay (at least, laboratory consolidation phenomena). Curvature of the compression curve on a semilogarithmic plot, therefore, is not necessarily an indication of minimum disturbance of the clay. In the first stage of consolidation, the loose open structure and the chains of clay grains have been compressed in a vertical direction only, and possibly have been folded down upon each other, with some disturbance to the bonds, to form a structure under increasing pressure consistent with the void ratio, but still characteristic in form. No such complete remolding can possibly occur as a result of disturbance by compression which will cause the material to assume the dispersed remolded structure. During compression the remolded clay has also acquired structural bonds between grains (thixotropic effects). Beyond the intersection of the curves, the structure of the undisturbed and remolded clay has approximately identical effects upon compression, which takes place approximately along the flatter slope of the remolded consolidation curve, although the two structures are not identical in form.

This phenomenon is clearly evident in the case of the Mexico City clay in Figs. 5(*a*) and 5(*b*), where a break in the consolidation curve for the undisturbed clay occurs in the region of 5 kg per sq cm, and thereafter follows approximately along the curve of the remolded clay. The writer has observed such curvature in the consolidation curves of many different types of clays, the break occurring more frequently under rather low pressures for soft organic clays. This curvature or break in the slope of the compression curve of undisturbed clays is only one of several stages in the ultimate consolidation of soft clay deposits to shale. These stages are evident only on the semilogarithmic plot of void ratio against pressure.

Another point that should be mentioned is that disturbance to the natural structure tends to make the clay less permeable and hence also tends to make the coefficient of consolidation too small. The actual rate of settlement,

therefore, will be more rapid than the estimates. The effect of disturbance cannot be detected in the form of the time-consolidation curves.

From the viewpoint of the application of the results of laboratory tests, the primary purpose of laboratory analyses is to furnish suitable representative averages for soil properties to be used in settlement and stability investigations. The writer has used a "Control Test Method" as an aid in selecting the least disturbed samples for test. Unconfined compression tests are made on all samples, as soon as received, to determine their quality, using prismatic specimens trimmed from the least disturbed central portion of the sample. It has been observed that the compressive strength of specimens, using specimens the full diameter of the sample (3-in. sampler or 2-in. seamless tubes), was 50% to 75% of the strength of small specimens obtained by trimming away the circumferential disturbed material. The ratio of the unconfined compressive strength, remolded to undisturbed, is used as a measure of the quality of the sample. This strength ratio should be less than some fraction, which is determined and revised as the testing progresses. For clays consolidated under light pressures, the ratio may be as low as $\frac{1}{8}$ to $\frac{1}{10}$, or less, depending on the character of the clay, but may be only $\frac{1}{2}$ to $\frac{1}{5}$ for clays consolidated under high pressures. However, this criterion alone is usually not sufficient, because its significance varies with the character of the clay. Frequently, the variations observed in the horizontal direction in the character of the clay of a given stratum are considerably less than those normally found in the vertical direction in this same stratum, due to changes in the geological and sedimentation conditions as the deposit was formed.

It has been observed frequently that the liquid limit, as a valuable measure of the character of the clay, increases consistently with depth, with a corresponding increase in the compression index in the consolidation test (a steeper compression curve denoting a clay that is more compressible) and with changes in the unconfined compressive strength and strength ratio, which may or may not be consistent. To provide a suitable and fairly reliable basis for judging the quality of the clays for the purpose of selecting test results and computing averages of properties from samples, which are judged to have suffered the least disturbance and to be most representative of subsurface conditions, the characteristics of the clay must be studied. Diagrams of soil properties are extremely useful for this purpose, and form a valuable part of any report of settlement or stability investigations. The diagrams should show the following:

(1) The variation with depth of the unconfined compressive strength, strength ratio (remolded to undisturbed), moisture content, liquid limit, and other test properties, if considered pertinent to the problem.

(2) The relationship between moisture content as a common argument and these same properties, in order to indicate the range of materials encountered and the characteristics of each type.

The writer believes that the triaxial compression test offers greater possibilities for the future than any other method of determining the shearing properties of soils. However, a direct application of test results cannot always

be made, such as in the case of the unconfined compression test for determining the bearing capacity of clays, because of testing difficulties, but primarily because engineers do not know what the lateral pressures are in different types of deposits in nature and how they vary as the foundation loads are increased from zero to their ultimate value. In the unconfined compression test, unknown lateral restraints also exist, the magnitudes of which depend on the capillary pressure developed and the expansive properties of the clay at the preconsolidation pressure. These lateral forces probably are quite different from those existing in the deposit, being isotropic in character and also probably varying in magnitude as the vertical pressure is increased during the test. The issue has been merely side-stepped in this case. It is believed that curvature of the stress-strain curve in the first 20% of the unconfined compressive strength is not necessarily an indication of disturbance for clays consolidated under light loads and with compressive strength less than about 0.5 kg per sq cm, because of the variation in the magnitude of the lateral pressures that may be developed as loading progresses.

The fact that clays consolidate during triaxial compression tests should not be considered as a disadvantage of the test. This must be accepted as the normal phenomenon of clays, which are in a natural state of consolidation under their present overburden pressure. The primary problems are to reestablish the natural state of stress on the sample that existed in the deposit, and then to increase the stresses on the specimen in such a manner as to resemble those imposed on the actual material in place. Although the preconsolidation pressure determined by a consolidation test may fairly represent the natural vertical stress conditions, the lateral pressure may be of the order of 50% to 75% of this value, depending on the type of clay and the geological and load history of the deposit. It may be possible to obtain some approximate information on the natural pressure ratio—vertical to lateral—from careful measurements of the stresses set up in the consolidation ring after equilibrium has been reached under each pressure increment in the consolidation test.

The important question is: Are the final results the same and are they equally representative for the following test procedures?

(*a*) The clay specimen is preconsolidated by suitable increments under an all-around pressure, σ_3, equal to some fraction of the vertical preconsolidation pressure. The vertical stress is then increased until failure occurs.

(*b*) The clay specimen is preconsolidated by suitable increments, maintaining some constant stress ratio, σ_1/σ_3, until σ_1 becomes equal to the vertical preconsolidation pressure. Thereafter, the vertical stress is increased until failure occurs.

In either case, to simulate rapid failure conditions in a thick clay deposit, a slow triaxial compression test should be run as a closed system with measurement of pore water pressures as the vertical stress is increased to failure. If special care is taken in setting up the test and running it, and a balanced pressure system is used for measuring pore water pressures at the top and bottom of the specimen, fairly reliable information may be obtained on the effective stresses acting during the test and at failure. Tests run at pressures

somewhat below and above the vertical preconsolidation pressure would provide a relation between the strength properties and void-ratio change to be used for estimating the natural strength properties. It is obvious that the triaxial compression test can never be reduced to the simple testing procedure and method of interpretation used in the unconfined compression test, and in this respect the unconfined compression test has a great advantage of simplicity. In the final analysis, it is the correlation with the actual behavior of full-scale structures that should determine the relative reliability of any test method.

Professor Rutledge has presented a very timely paper on this subject and has opened up a valuable field for further research.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

RÔLE OF THE LAND DURING FLOOD PERIODS

Discussion

BY MESSRS. RAPHAEL G. KAZMANN, AND W. G. HOYT AND W. B. LANGBEIN

RAPHAEL G. KAZMANN,[12] JUN. AM. SOC. C. E.[12a]—The material presented by Mr. Horner is a valuable addition to quantitative hydrology. Nevertheless, it should be pointed out that, throughout the paper, there is a misleading use of the word "porosity." The porosity of a material is no indication of its ability to transmit water through its interstices. The ability of a material to function as a "conduit" for the transmission of water generally is known as the material's permeability.

As used by the Geological Survey, U. S. Department of the Interior, the coefficient of permeability of a water-bearing material is defined as the rate of flow of water, in gallons per day, through a cross-sectional area of 1 sq ft under a hydraulic gradient of 100% at a temperature of 60° F.

The porosities of water-bearing materials bear little relationship to their permeabilities. This is illustrated by Table 7, adapted from *Water-Supply Papers Nos. 596-F* and *887*[13,14] of the Geological Survey. Table 7 demonstrates that the water-transmitting properties of materials generally are not related directly to their porosities. The materials listed in Table 7 were obtained in various sections of the country and at various depths below the land surface. The coefficients of permeability were determined in the laboratory by passing water through samples of the material by means of techniques which are described in detail in *Water-Supply Paper No. 887*.[15] It may be concluded, therefore, that the term "porosity," as used in Mr. Horner's paper, should be replaced by the term permeability, or by another term expressing the same concept.

NOTE.—This paper by W. W. Horner, M. Am. Soc. C. E., was published in May, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1943, by L. K. Sherman, M. Am. Soc. C. E.

[12] Asst. Hydr. Engr., Ground Water Div., U. S. Geological Survey, Washington, D. C.

[12a] Received by the Secretary June 9, 1943.

[13] "Laboratory Tests on Physical Properties of Water-Bearing Materials," by N. D. Stearns, *Water-Supply Paper No. 596-F*, U. S. Geological Survey, 1928, pp. 164–169.

[14] "Methods for Determining Permeability of Water-Bearing Materials," by L. K. Wenzel, *Water-Supply Paper No. 887*, U. S. Geological Survey, 1942, pp. 13–14.

[15] *Ibid.*, pp. 50–71.

Further, Mr. Horner's paper could be strengthened by including a treatment of the movement of ground water. In dealing with the problem of subsurface retention and detention, there is no reference to the rise and fall of the water table during or following any of the storms; nor does any attempt

TABLE 7.—POROSITIES AND PERMEABILITIES OF WATER-BEARING MATERIALS

Description of material (1)	Porosity (%) (2)	Coefficient of permeability (3)	Description of material (1)	Porosity (%) (2)	Coefficient of permeability (3)
Terrace gravel	23.6	1,230	Glacial gravel	38.0	90,000
Gravel mixed with some coarse sand	24.1	4,500	Washed gravel	38.5	10,500
Gravel	25.1	20,000	Red clay	42.4	20
Alluvium gravel (gravel and silt)	29.3	70	Sand	44.4	1,100
Sand and gravel	29.7	1,820	Sand	44.8	560
Sand	35.8	2,600	Silty sand	45.1	6
Fine gravel, coarse sand mixed with clay balls	37.0	30	Sand	45.8	410
			Loess	53.2	7
			Subsoil	54.2	20
			Subsoil	55.5	0.2

appear to have been made to obtain data of this sort. An analysis of such data would have been a valuable product of the investigation, since any major addition to subsurface detention would have an appreciable effect on both flood flows and normal flows into streams.

W. G. HOYT,[16] M. AM. SOC. C. E., AND W. B. LANGBEIN,[17] ASSOC. M. AM. SOC. C. E.[17a]—With the passage of the Flood Control Act of June 22, 1936, an obligation was imposed on the engineering and associated professions to determine the magnitude of reduction in flood stages and discharges that could be effected through changes in land use. The author describes a technique for evaluating the rôle of land during flood periods with no snow or ice, and limited to areas where (see heading, "Part I.—Introduction: Engineering Technique") "Infiltration capacity is small to moderate * * *, and stream-channel inflow occurs predominantly as surface runoff," a situation which the author indicated "applied quite widely to cultivated and grass lands."

Through the use of infiltration-capacity curves for small experimental catchment areas, homogeneous in soil, cover, and cultural treatment, surface runoff volumes as the result of changes in land use are determined, and finally the runoff volumes are translated into river stages and discharges. To the extent that flood rises caused by rain are composed of surface runoff only and to the extent that "correct seasonal average infiltration curves" can be derived and placed "in their proper position on the rainfall patterns," the method enables one to evaluate a reduction in surface runoff through changes in land use. Such quantitative determinations are necessary if the feasibility of flood control through treatment of land is to be evaluated.

Flood control operations of any nature involve the expenditure of millions of dollars, and it is essential that the degree of control to be effected should be

[16] Hydr. Engr. (Prin.), Water Resources Branch, U. S. Geological Survey, Washington, D. C.
[17] Associate Hydr. Engr., U. S. Geological Survey, Washington, D. C.
[17a] Received by the Secretary August 9, 1943.

well demonstrated. The hydraulics of flood control through the use of engineering structures can be evaluated by verifiable and comparatively well-founded technical processes. The evaluation of the hydraulics of land use during flood periods is accomplished by techniques that are quite complex and speculative. This is true because of (1) a lack of basic knowledge concerning the land phase of runoff, (2) wide variations in infiltration capacity and other hydrologic conditions within a drainage basin, and (3) a lack of demonstrated beneficial change in flood-runoff régime of any large stream because of a change in land use.

The author attempts to supply the first deficiency and is to be congratulated for breaking down[18] the process by which rainfall is translated into runoff, for outlining the manner in which meager observational data may be used to give quantitative expression to the factors involved, for computing the runoff for a typical drainage basin in Texas in accordance with the concepts outlined, and for comparing it storm by storm with the measured volumes and rates. This comparison between computed and observed results may be necessary, but is it sufficient to prove the correctness of the postulates? The derived relations may furnish good empirical working tools, but the writers suggest that other postulates may evolve equally good checks.

The author presents curves of infiltration capacity for soils typical of the Texas Blacklands that show marked differences in position, and, therefore, for a given rainfall pattern and antecedent conditions, marked differences in surface runoff for different kinds of land use These curves are based on measurements of runoff on small homogeneous catchment areas at the Waco watershed and the Garland Runoff Studies Project. There is no reason to believe that these curves do not reflect the qualitative effects of cover on the surface runoff of the small homogeneous areas for which they were derived. There is nothing in the report, however, to demonstrate that a river such as the Trinity River would respond to changes in land use in the same way as the small homogeneous plots. Thus, although the author has devised a technique which will show quantitative difference in surface runoff (the immediate task before him), there is still lacking a process for evaluating the effect of land-use change on flood stages and flows which is not open to considerable doubt.

Some may feel, as do the writers, that the author has placed too much dependence on the concept of "infiltration capacity," arrived at, as it is, indirectly through the measurement of surface runoff on small experimental areas in a procedure designed to simulate flood rises from large natural drainage basins under assumed combinations of land use.

In so far as the engineering technique is concerned and as the author quite clearly implies, an expanded title for the paper might well read, "Rôle of the Land During Flood Periods in Those Areas Where Infiltration Capacity Is Low to Moderate, Where Infiltrated Water Does Not Appear in the Stream as 'Quick-Subsurface Return Flow,' and Only During Periods of No Snow, Ice, or Frost." These restrictions in the use of the technique impose quite serious limits on described procedures from the viewpoint of practical application to

[18] "See also "Hydrology," Physics of the Earth—IX, edited by Oscar E. Meinzer, McGraw-Hill Book Co., Inc., New York, N. Y., 1942, pp. 507–513.

general flood problems. They do not necessarily detract from the author's conclusion with respect to the East Fork of the Trinity River provided that its basin characteristics can be definitely determined to be such that the foregoing limitations apply. Apparently, at one time, conditions in the Trinity River basin were such that the author's technique would not apply. It is stated in the final report on the Trinity River watershed:[19]

> "There is evidence that floods occurred on the Trinity before settlement but they were vastly different from the present floods. Water rose and receded so slowly that many streams which are now intermittent then flowed all year."

This description implies an area with large infiltration capacity, much ground-water storage, little surface runoff, and a slow release of infiltrated water to the stream as sustained flow—conditions that in the main are seemingly inherent to a basin and not modified so materially by human occupation.

The writers know of no simple method to determine, on an areal basis, whether subsurface inflow constitutes less than 10% of flood volume. Although the author may be correct in his assumption that 90% of the present flood volume in the East Fork of the Trinity River is surface runoff, no basic data have been submitted to prove it. The total runoff (surface) at the Rockwall gaging station for the thirty-five storms, May 3, 1935, to July 3, 1940, as reported in Table 4 is 31.29 in. The total flow past the Rockwall gaging station for the same period is about 40 in. In other words, 78% of the total runoff that appeared at Rockwall is assumed by the author to be surface runoff. The difference between these two amounts (8.7 in.), except for a few flood rises not included and which approximate 0.70 in., is presumed to represent all the infiltrated water that reached the stream channel at Rockwall.

Based on an extensive knowledge of hydrologic conditions at the North Appalachian Experimental Watershed near Coshocton, Ohio, a research project of the Hydrologic División, Office of Research, Soil Conservation Service, U. S. Department of Agriculture, H. S. Riesbol, Assoc. M. Am. Soc. C. E., concluded[20] substantially as follows: Extensive study of ground-water levels, percolation rates, hydrograph recessions, and normal ground-water depletion curves should lead to dependable and practical knowledge concerning when and where infiltrated water joins channel flow as seepage and spring flow from perched water tables, temporary zones of saturation, or from deep seepage. No such basic information is submitted for the Trinity River basin.

The writers do not question the accuracy of the various infiltration-capacity curves shown in Fig. 5 in which infiltration capacity is shown to increase progressively from untreated farm land to well-managed forests. Through the use of such a family of curves under any given rainfall pattern, computed reductions in surface runoff as a result of betterment in cover or land use are obtained. Whether or not these computed reductions in surface runoff can be assumed to be

[19] *H. R. Doc. No. 708*, 77th Cong., 2d Session (letter from the Secretary of Agriculture transmitting a report of a survey of the Trinity River watershed based on an investigation authorized by the Flood Control Act of June 22, 1936).

[20] "Some Aspects of Subsurface Water in Hydrologic Research Agricultural Watersheds," by H. S. Riesbol, *Proceedings* of the Hydrologic Conference, Pennsylvania State College, State College, June 30 to July 2, 1931, p. 107.

equivalent to reduction in the flood runoff of the stream draining the particular area, however, seems open to question. The writers believe the author fully understands this situation, but he fails to delineate or give an expression to the area where his technique would apply except the broad statement (see heading, "Part I.—Introduction: Engineering Technique") that the "situation applies quite widely to cultivated and grass lands." It would seem quite the contrary with respect to much of the cultivated land in the United States since the agriculturist would tend to select, among other factors, soils that combine high infiltration capacities with satisfactory moisture-retention properties. The real criterion underlying the author's technique is not so much whether the cover is cultivated or in grass but rather, the subsurface storage characteristics. Will the subsurface storage spaces retain the infiltrated water, or are they of such limited extent that infiltrated water passes into stream channels where it forms a part of the direct flood runoff of all stream rises?

The author suggests that the ratio between observed surface runoff and computed rainfall excess is a test of the validity of his procedure. It is stated that for five storms the rainfall data were inadequate and that measured runoff was used as a guide in the determination of rainfall excess. Examination of the author's unpublished reports on his study as a consultant to the Department of Agriculture indicates that the computation of rainfall excess in the other thirty storms was influenced to a considerable extent by knowledge of the actual volume of runoff as measured at the gaging station maintained by the U. S. Geological Survey at Rockwall. Some of these influences included reinterpretations of the rainfall patterns, adjustments in the position of infiltration-capacity curves, and deviation from the seasonal M-curve values. The correction factor (Col. 6, Table 4) therefore reflects the errors that remain after all readily explainable adjustments have been made. It is obvious that wide departures from observed runoff have been adjusted in order to arrive at a result more closely in conformance with observed runoff. The test is not the same as would be met by a forecaster who has no knowledge of the runoff to be produced by the storm other than that provided by his working formulas. If partial advantage is taken of the runoff as it was, why not take full advantage? The technique might not be so different. If the author had wished, he might have continued his adjustments until conformance of computed and observed runoff was obtained. Perhaps a method varying the M-curve or other factor to this end can be devised. Whether or not the author's procedure can be considered practicable for ordinary engineering practice will depend on the man-hours involved in its application. Some idea as to the number of man-hours required to simulate the thirty-five flood peaks would be enlightening.

An examination of Fig. 5 and a study of the method by which the family of infiltration-capacity curves was placed on the rainfall graphs suggests that the results are highly sensitive to rainfall intensity. Hourly rainfall amounts were estimated from data so scant as to be almost nonexistent; but, assuming that the hourly pattern shown in Fig. 5 is correct, minute-to-minute variations in intensity necessarily have been neglected. For example, consider the hour

interval, 1 p.m. to 2 p.m., during which there was 1.56 in. of rainfall: Suppose 0.20 in. of this amount fell during the first 20 min, 0.52 in. during the middle 20 min, and 0.84 in. during the last 20 min; then the amount of mass infiltration that occurred prior to rainfall reaching the infiltration capacity would be increased by 0.20 in., and the initial infiltration capacity on curve (1), Fig. 5, would be 0.80 in. per hour instead of 1.09. The net result would be that the rainfall excess during this hour would be 1.03 in. for cultivated untreated land and 0.91 in. for strip-cropped, both important land classes. The author's values are 1.09 in. and 0.92 in., respectively. It is significant that, by assuming a different rainfall distribution, the computed reduction in runoff is less. The author's difference between the two values is 0.17 in.; that computed for a possible distribution of rainfall assumed by the writers is 0.12 in. Seemingly, the effect of any error that would advance the period of intense rainfall would increase the computed difference between "before" and "after"; an error that would retard the period of intense rainfall would diminish the effect.

The report explains the influence of antecedent conditions as they affect soil moisture on the infiltration capacity during a storm. Increasing the volume of infiltration prior to a storm decreases the capacity of the basin to retain water. Therefore, under the changed conditions, to what extent would the presumed increase in infiltration operate adversely to lower infiltration capacities during subsequent storms? Moreover, are there not economic implications of the reduction in flood runoff, particularly with respect to the rights of downstream riparian owners or appropriators? In this particular case, using the values given in Col. 13, Table 4, the 20.4% reduction in total surface runoff means a reduction of 16% in total stream flow.

The annual runoff in inches at the Rockwall gaging station is greater than that at other stations in the upper Trinity River basin.[21] It is much greater than at any of the basins at the Waco Experiment Station.[22] Does this difference in annual runoff indicate difference in land types or land use or does it conform to differences in rainfall?

Query is raised as to the use of the term "regression curves." Apparently this term as used refers to the die-away curves of infiltration capacity shown in Fig. 5. Lines or curves of "regression" are statistical terms devised by Sir Francis Galton[23] to describe lines or curves of best fit between a single independent variable and one or more dependent variables. In this sense the die-away curves of infiltration capacity may be regression curves; but, then, so are all the other empirical lines or curves shown. A more descriptive term should be devised.

The *M*-curve shows that the spring season overlaps the adjoining winter and summer seasons. This is understandable, but what procedures are used to determine which seasonal curve shall govern in a particular condition?

[21] "Surface Water-Supply of the United States," *Annual Reports*, U. S. Geological Survey, 1924–1942.

[22] "Hydrologic Data, Blacklands Experimental Watershed, Waco, Tex.," *Hydrologic Bulletin No. 2*, U. S. D. A., SCS, 1942.

[23] "An Introduction to the Theory of Statistics," by G. Udny Yule, C. Griffin and Co., Ltd., London, 1924, p. 176.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SLUDGE DRYING DEVELOPMENTS AT CHICAGO, ILL.

Discussion

BY C. W. GORDON

C. W. GORDON,[2] ESQ.[2a]—In presenting this paper Mr. Johnson has made available a wealth of design data, and he has discussed operating experiences frankly. This procedure gives the complete picture, and as such will be a great benefit to design and operating engineers. The Chicago Sanitary District was the pioneer in the field of sludge drying and incineration, and its method of sludge disposal is recognized as a success by sanitary engineers everywhere.

During 1940, multiple-hearth plants dried and incinerated 215,912 tons of filter cake. This is equivalent to 70,861 tons of dry solids. Since multiple-hearth plants are not generally designed for fertilizer production, only a small amount was produced on a casual basis.

A total of 651,869 tons of filter cake equivalent to 155,155 tons of dry solids was disposed of by drying and incineration in the United States during 1940. These data do not include the fertilizer production from rotary drier plants. Operations at Milwaukee, Wis., will make an impressive addition to this total. The production at Pasadena, Calif., Toledo, Ohio, Dayton, Ohio, and Grand Rapids, Mich., to mention only a few of the others, will undoubtedly swell the total dry-solids production to more than 200,000 tons.

At first glance one may well ask the question, "Is not this system unnecessarily complicated?" Taking into consideration the necessary functions will show that this is not the case.

First of all, "What is the problem?" The problem is to start with dewatered sludge ranging from 60% to 90% initial moisture, evaporate this moisture, burn the dry solids remaining, remove the odors and fly ash from

NOTE.—This paper by Lloyd M. Johnson, Esq., appears on pp. 1015–1023 of this issue of *Proceedings*.

[2] Chf. Engr., Drying and Incineration Dept., Raymond Pulverizer Div., Combustion Eng. Co., Chicago, Ill.

[2a] Received by the Secretary June 28, 1943.

the gases discharged to atmosphere, and pump the total ash to fill. The final products are then odor and dust-free gases and a sterile ash.

When only fertilizer is being produced, the problem is somewhat simplified. The problem then is to begin with dewatered sludge, evaporate the moisture, remove the odors from the gases discharged to the atmosphere, and transport the fertilizer to storage. The final products in this case are odorless gases and a valuable fertilizer. Thus in producing fertilizer the problem of ash disposal is eliminated. For large operations, such as at the West-Southwest works in Chicago, this is an important factor, since dry sludge normally contains at least 50% ash. Thus, when 300 tons of dry solids are incinerated per day, 150 tons of ash remain for disposal. This ash is quite abrasive, and the maintenance of a large ash handling system is no small item.

Each installation requires individual study, and economic considerations should govern the decision. When there is a question regarding fertilizer production, the better solution is to select a system which can produce fertilizer or incinerate with equal efficiency and without reduction in capacity.

The value of the finished fertilizer will depend upon the origin and treatment of the sludge. Activated sludge, as produced at Chicago and Milwaukee, contains 6% nitrogen, whereas digested sludge contains about 2.5% nitrogen. This is not the entire story, since dry sludge fertilizer is a complicated compound; aside from being an excellent soil conditioner, it has numerous positive fertility values. The Research Department of the Chicago Sanitary District, through an extensive test program, has proved that its flash-drying system completely destroys all pathogenic organisms in the sludge. Thus it is a safe fertilizer.

To combine the functions of steam power generation with sludge drying and incineration is fully justified for large operations such as at the Southwest works. For smaller operations, the combination becomes less attractive, particularly when power is available at reasonable rates. The operation of the equipment is essentially automatic and does not require constant attention. For example, in one plant serving an equivalent population of 100,000, one operator supervises both the vacuum filters and the incinerator with entire satisfaction. This is possible in a properly designed plant when the filters and incinerator have a common operating floor.

This paper mentions abrasion at several points in the flash-drying system. Abrasive action at the points indicated is unavoidable. Experience at the Southwest works and all other plants indicates that maintenance costs from this source are very nominal, especially when related to the dry solids production of the plant. For example, a set of mixer paddles will last from one year to three years, a steel cyclone lining from one year to five years, and a set of cage bars in the flash drier from one year to three years. These wide variations in life show the effect of silica content of the sludge. Obviously, it is desirable to remove all the silica in grit chambers.

Mr. Johnson has stressed the problems of the operating staff. This is perfectly natural for a front-line officer. To obtain the true perspective, it

is necessary to balance results against costs and operating difficulties. The results speak for themselves at the West-Southwest works. Taking into account the fact that this is the largest sewage treatment and disposal plant in the world, together with the fact that this is the first time steam power generation has been combined with sludge drying and incineration, the accomplishments reported in this paper are amazingly good.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

FLOW AROUND BENDS IN STABLE CHANNELS

Discussion

BY MESSRS. J. M. ROBERTSON, AND JOE W. JOHNSON AND M. A. SELIM

J. M. ROBERTSON,[15] JUN. AM. SOC. C. E.[15a]—The conclusions reached in this paper appear to be in essential agreement with those of previous investigators. For instance, the author states (see Conclusion 2) that "The downstream components of the filamental velocities in the first half of the bend are greater near the convex bank than at the concave bank * * *." Essentially the same conclusion was reached by the late D. L. Yarnell,[16,17,18] M. Am. Soc. C. E., at the University of Iowa at Iowa City, in connection with his extensive study of the flow of water around bends in open and closed channels. In the paper in which he and Mr. Woodward report on the flow in open and closed 180° bends the following conclusions are reached:[17] "* * * with substantially uniform velocity distribution in the approach channel to the bend, the velocities of the filaments along the inside wall of the bend are increased while the velocities of the filaments along the outside wall are reduced, * * *" and, "as the water flows around a bend the effect of wall friction is to produce an unstable relation between the filaments flowing at different velocities, with the result that the filaments of highest velocity tend through a secondary circulation to drift toward the outer wall of the bend." This second statement pertains to the flow near the end of a 180° bend. In spite of information such as this, the writer can refer to several well-known books on hydraulics in which data are presented and statements are made, inferring that, as water flows around a bend, the filament of maximum velocity is always at the outside of the bend. Actually, as may be seen from the author's study, the filament of maximum velocity is initially, and, for most of the way around the bend, near the inside of the bend.

NOTE.—This paper by C. A. Mockmore, M. Am. Soc. C. E., was published in March, 1943, *Proceedings*.

[15] Asst. Prof., Dept. of Eng. Mechanics, Pennsylvania State College, State College, Pa.

[15a] Received by the Secretary June 23, 1943.

[16] "Flow of Water Around Bends in Pipes," by David L. Yarnell and Floyd A. Nagler, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1018.

[17] "Flow of Water Around 180-Degree Bends," by D. L. Yarnell and S. M. Woodward, *Technical Bulletin No. 526*, U. S. D. A., October, 1936.

[18] "Flow of Water Through 6-Inch Pipe Bends," by D. L. Yarnell, *Technical Bulletin No. 577*, U. S. D. A., October, 1937.

The flow of water or any fluid around a bend in an open or closed channel is affected by many factors; it is unfortunate that Professor Mockmore did not investigate more of them. Besides the primary variable of the rate of flow (or the Reynolds number) the flow conditions in a bend should depend upon the relative curvature of the bend, the shape of the channel cross section, and the length of the bend. Also, when one bend follows another, the flow conditions in the second bend are modified greatly by the relative proximity of the first bend. The writer will attempt to evaluate the effect of some of these variables on the flow in open channel bends through reference to data obtained mainly in closed conduit bends. If one keeps in mind that, at least in most respects, the flow in an open channel is similar to that in the lower half of a closed channel of twice the depth, it is not difficult to translate these observations to open channel bends, at least qualitatively.

In one part of his mathematical analysis the author assumes that the downstream component (w-component) of the velocity follows the free vortex law, according to which the velocity at any point is inversely proportional to the radius to that point. As was shown by Fig. 15, this is a good assumption. It would be expected from this law that the sharper the curvature of the bend the more pronounced would be the velocity distribution. Several years ago in connection with a study of the flow of water around bends in pipes at the Hydraulics Laboratory of the University of Illinois at Urbana, under the direction of W. M. Lansford, M. Am. Soc. C. E., the writer measured the velocity distribution in several 90°, cast-iron, flanged, bends in a 4-in. pipe line.[19] The bends were preceded by some 450 diameters of straight pipe, thus assuring a normal velocity distribution above them; and the measurements were made across a diameter, 45° around the bend, in the plane of the bend. Using the ratio $\frac{r_c}{b}$ (r_c is the radius of the center line of the bend, and b is the channel breadth or pipe diameter) as indicative of the relative curvature of the bend, the curvatures of the bends studied were 0.8, 1.0, 1.5, and 2.0. Two bends of each curvature were studied. Typical velocity distributions obtained are shown in Fig. 17(a) for a mean flow velocity V of about 4.8 ft per sec. The effect of curvature on velocity distribution is quite apparent. A rough comparison of the results obtained with the free vortex law prediction is shown in Fig. 17(b) for two rates of flow in the bends of least and greatest curvatures. The constant in the free vortex law relation for each case was chosen arbitrarily so that

$$w\,r = k = V\,r_c \qquad (31)$$

The general trends predicted by the free vortex law appear to be verified, but the actual distributions are modified greatly due to the effect of the velocity distribution in the approach pipe. In general, it appears that the sharper the curvature the greater will be the maximum velocity and the nearer the filament of maximum velocity will be to the inside of the bend.

Another case in which the curvature plays an important rôle is in connection with the superelevation of the water surface as the water flows around the

[19] "A Study of the Flow of Water Around Bends in Pipes," by J. M. Robertson, thesis for the degree of Bachelor of Science in Civil Engineering, College of Eng., Univ. of Illinois, Urbana, 1938.

bend. As the author states, this is due to the centrifugal action of the water rounding the bend. In a closed conduit this phenomenon manifests itself in a difference in pressure between the inner and outer curves of the bend or elbow.

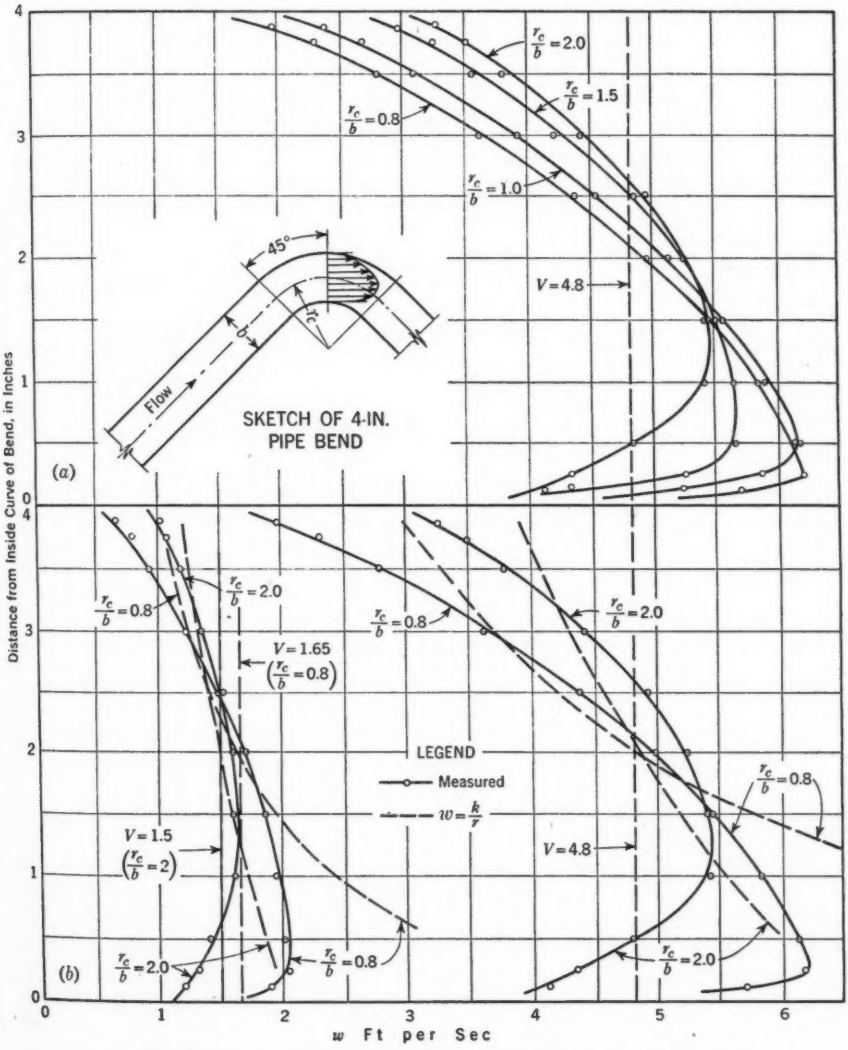

FIG. 17.—EFFECT OF CURVATURE ON VELOCITY DISTRIBUTIONS IN PIPE BENDS

As has been shown by Professor Lansford[20] and others,[16,17,21] this difference in pressure allows the bend to be used as a flow meter. Starting with the basic

[20] "The Use of an Elbow in a Pipe Line for Determining the Rate of Flow in the Pipe," by W. M. Lansford, *Bulletin No. 289*, Eng. Experiment Station, Univ. of Illinois, Urbana, 1936.

[21] "The Use of Pipe Bends as Flow Meters," by Herbert Addison, *Engineering* (London), March 4, 1938, pp. 227–229.

force relation given by Eq. 1, the expression for this difference in pressure Δh, in feet of fluid flowing, in terms of the velocity head $\frac{V^2}{2g}$ in the pipe line (V is the average velocity) becomes,[6,20]

$$\Delta h = \frac{2b}{r_c}\frac{V^2}{2g} = K\frac{V^2}{2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

For a circular pipe, b is the diameter; for a rectangular channel, it is the breadth. In the study on the flow in 4-in. pipe bends previously referred to,[19] the writer verified the relation between the coefficient K in Eq. 32 and the bend curvature. The results are shown in Fig. 18. The pressure differences all were measured

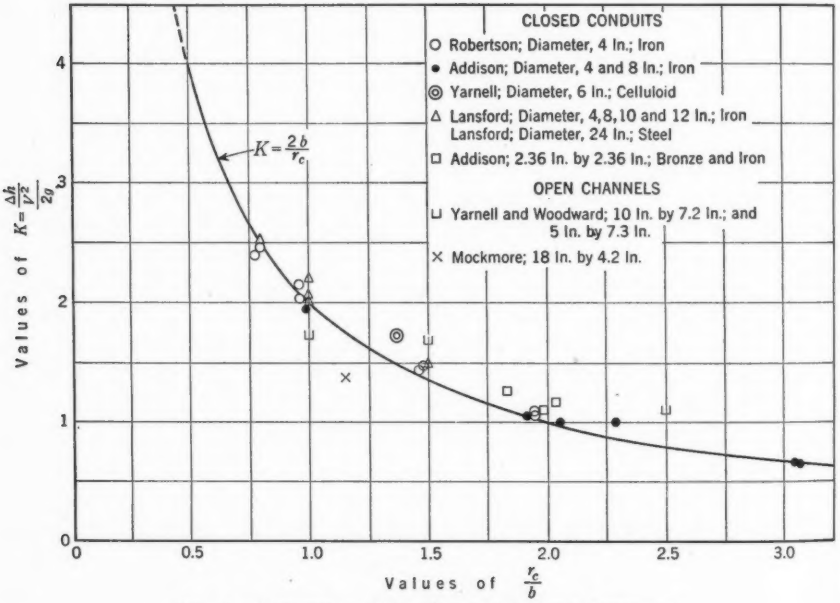

FIG. 18.—VARIATION IN BEND COEFFICIENT WITH CURVATURE

across a diameter 45° around the bend in the plane of the bend. The values shown for K were found to apply for velocities from 2.5 to 9.5 ft per sec. Below 2.5 ft per sec the coefficient increased somewhat, probably due to a Reynolds number effect such as is observed for other differential head meters. Also shown on this plot are data taken by Herbert Addison[21] for 4-in. and 8-in. circular cast-iron pipe bends, by Mr. Yarnell[18] for a circular 6-in. celluloid bend, by Professor Lansford[20] for various sizes of bends up to 24 in., and by Mr. Addison[21] for two cast-iron and one brass, square (b = 6 cm), closed channel bends. Considering the fact that all of these elbows and bends may not have had such ideal approach conditions as did the writer's, the agreement is excellent.

[6] "Hydraulics of the Miami Flood Control Project," by S. M. Woodward, *Technical Reports*, Miami Conservancy Dist., Dayton, Ohio, Pt. VII, 1920, p. 264.

The effect thus shown of the curvature on the pressure difference between the inside and outside curves of a bend should apply equally well to the superelevation of the water surface around a bend in an open channel. From the data presented by Messrs. Yarnell and Woodward[17] on the flow around 180° open and closed rectangular channels, the writer has computed the coefficient K for two different open channel bends. The computations were made from plotted results given for conditions 45° around a 10-in. and a 5-in. wide channel. These data are plotted in Fig. 18, and the agreement with the pipe data is seen to be fair. Finally, the writer has estimated a point from data in Fig. 2(*a*) by means of considerable interpolation. From this figure at the start of the bend (Section 8) Δh is 0.25 in. and V is 0.898 ft per sec, and at 90° around the bend (Section 12) Δh is 0.13 in. and V is 0.832 ft per sec. Assuming the average of these values to be those which apply at a station 45° around the bend, a value of K equal to 1.36 is obtained. For this bend $\frac{r_c}{b}$ is 1.16 and K should be 1.7. In view of the assumptions made this is probably a fair check.

As to the other factors which affect the flow conditions around bends in open and closed channels, the writer has no data of his own to quote. However, he would like to mention the observations of others which may be pertinent to the discussion in connection with the effect of shape of the channel on flow conditions. Mr. Yarnell[18] notes that "the strength of the secondary currents in bends is greater in wide channels than narrow channels." In an excellent discussion[22] of the flow around curved channels and the occurrence

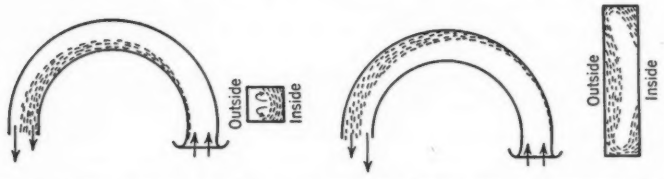

FIG. 19.—FLOW CONDITIONS IN SQUARE AND DEEP RECTANGULAR SECTIONED BENDS

of secondary currents, reference is made to some observations of A. Betz[23] on the flow in rectangular channels of different shapes. Fig. 19 indicates the nature of the flow observed in a square and in a narrow rectangular channel showing the regions in which an accumulation of retarded fluid tends to occur. In each case the plan view depicts conditions in the middle of the section and the sectional view conditions at the end of the bend. The explanation for the conditions in the square section are rather simple. As many observers have noted, because of the secondary motion the faster moving fluid in the central part is moving outward, pushing the fluid in the boundary layer at the outer wall to the top and bottom of the channel and thence back toward the inner wall of the bend where it accumulates.

[22] "Modern Developments in Fluid Dynamics," edited by S. Goldstein, Oxford Press, Vol. 1, 1938, pp. 84–87.

[23] "On Turbulent Frictional Layers at Curved Walls," by A. Betz, Vorträge aus dem Gegiete der Aerodynamik und verwandter Gebiete, Aachen 1929, A. Gillis, L. Hopf, and Theodor von Kármán, editors, Berlin, Julius Springer, 1930, pp. 10–18.

In a narrow, deep curved channel almost the reverse seems to occur as shown in the right side of Fig. 19. Here the secondary currents are confined to the top and bottom parts of the section where they tend to act just as in the square channel. The middle part of the section is too far from the top and bottom for the secondary currents to affect it greatly. The effect of the centrifugal forces on the fluid elements in this region tends to accentuate the retarded layer at the wall.[22] Although this type of action probably rarely occurs in open channels as they are more generally wide and shallow, the explanation given tends to show why (as Mr. Yarnell noted) the secondary currents are stronger in wide channels. Reference should also be made to the observations of V. M. Hegly[24,25] on the flow around a bend in a model canal of compound trapezoidal section. This canal consisted of a deep section and an adjacent shallow section of about one third the depth and about the same width. The velocity distributions are shown to be considerably different when the deep, instead of the shallow, region is against the concave bank.

JOE W. JOHNSON,[26] ASSOC. M. AM. SOC. C. E., AND M. A. SELIM,[27] JUN. AM. SOC. C. E.[27a]—A mathematical analysis of the problem of flow around a bend has been presented in this paper. Some rather extensive and elaborate experimental measurements also are given. The velocity distribution included in the experimental data is sufficiently complete to permit the determination of the velocity-head correction factor α and the momentum factor β,[28] although these terms apparently have not been computed by the author.

From the velocity distributions given in Fig. 2(*a*), values of α and β have been computed and are summarized in Table 3(*a*). These data show the variation of α and β with the location of the various sections. Although not conclusive, the values of the coefficients appear to be highest at the entrance end of the bends and to be least at the exit end.

TABLE 3.—VALUES OF α AND β

Item	(*a*) VARIOUS CROSS SECTIONS						(*b*) SECTION 12		
	Station 8	Station 12	Station 16	Station 20	Station 24	Station 28	Fig. 2(*b*)	Fig. 2(*a*)	Fig. 16
Stage	0.406						0.030	0.406	0.496
Q	0.449 (see Fig. 2(*a*))						....	0.449	0.449
α	1.12	1.13	1.05	1.62	1.37	1.15	1.46	1.13	1.31
β	1.05	1.11	1.02	1.25	1.14	1.04	1.16	1.11	1.14

[24] "Note sur l'Écoulement de l'Eau dans un canal à Profil Complexe," by V. M. Hegly, *Annales des Ponts et Chausées*, Mémoires et Documents, Vol. 106, No. 5, 1936, pp. 445–528.

[25] "Flow in Earthen Canals of Compound Cross-Section," by V. M. Hegly, translated and abstracted by Chilton A. Wright, Abridged Translations of Hydraulic Papers, *Proceedings*, Am. Soc. C. E., November, 1937, p. 36.

[26] Asst. Prof., Dept. of Mech. Eng., Univ. of California, Berkeley, Calif.

[27] Irrig. Dept., College of Eng., Fouad First Univ., Giza, Egypt; formerly Instr. in Mech. Eng., Univ. of California, Berkeley, Calif.

[27a] Received by the Secretary July 7, 1943.

[28] "Velocity-Head Correction for Hydraulic Flow," by M. P. O'Brien and J. W. Johnson, *Engineering News-Record*, August 16, 1934, p. 214.

Another item of interest that could be obtained from the various velocity distributions is a comparison between the values of α and β for a particular section at various stages and discharges. Data from the velocity distributions shown in Figs. 2(*a*), 2(*b*), and 16 for Section 12 are summarized in Table 3(*b*). Unfortunately, the data for only three stages are not enough to warrant any conclusions.

Values of the α and β coefficients for all the velocity distributions observed by the author might show an interesting trend of the data.

Of interest in this latter respect are the values of α observed in a straight flume at the U. S. Waterways Experiment Station.[29] Observation was by pitot tube on the velocity distribution at a section near the center of a flume 48 ft long and 2.31 ft wide, with various depths and slopes. In all observations the water-surface slope was parallel to the bed slope. The values of α for various runs are summarized in Table 4.

TABLE 4.—VALUES OF α FOR A STRAIGHT FLUME

Slope	Depth of Flow, in Feet					
	0.8	0.6	0.4	0.3	0.2	0.1
0.0002	1.037	1.068	1.060	....	1.068	....
0.00045	....	1.033	....	....	....	....
0.0005	....	....	1.040	....	1.052	1.045
0.00010	....	....	1.048	1.056	1.063	1.048
0.00015	....	....	1.054	1.093	1.061	1.089
0.00020	....	....	....	1.057	1.071	1.072
0.00040	....	....	....	....	1.057	1.063

Examination of Table 4 shows that, for a particular slope, α tends to increase slightly with decreasing depth. Actually, the range in variation is so slight that for practical purposes the average of the values for all depths and slopes represents a reasonable value of α for a straight flume. Thus, in a reach of a channel with no bends or obstructions the values of α, for most practical purposes, can be assumed equal to 1.

[29] "The Effect of Geometric and Slope Distortion on the Distribution of Energy and Tractive Force in Stream Cross-Sections," *Technical Memorandum No. 32-2*, U. S. Waterways Experiment Station, Vicksburg, Miss., 1936.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

FLOW CHARACTERISTICS AT RECTANGULAR OPEN-CHANNEL JUNCTIONS

Discussion

BY MESSRS. G. H. HICKOX, J. C. STEVENS, AND C. J. POSEY

G. H. HICKOX,[6] M. AM. SOC. C. E.[6a]—By attacking a problem which has received comparatively little attention Mr. Taylor has made a distinct contribution to engineering literature. His attempt at a solution points the way to a better understanding of the general subject of branching flow. A similar problem exists in pipe flow where several attempts have been made to predict the discharge through the orifices of irrigation sprinkler systems. Some of these attempts have been made on the basis of the momentum principle, relating the change in pressure in the feeder pipe to the rate of change of momentum. As far as the writer is aware, however, the pressure differences on the walls of the lateral outlets, parallel to the flow in the feeder pipes, have not been taken into account. The author's method of attack, considering the effect on the momentum equation of the wall pressures in the branch channel, is suggestive of similar treatment of the problem in pipe flow.

With respect to the application of the momentum principle to the problem of combining flow, Eqs. 1 and 2 state the sums of pressures and rates of momentum change correctly. The derivation of Eq. 3 from Eq. 1, however, is of somewhat doubtful validity. It assumes that the depths in the main channel and the branch channel at their junction are the same and are uniform across the channel. Also, it assumes that the pressure on the wall of the branch channel is equal to the hydrostatic pressure of still water at the same depth. This would be the case if the flow in the branch channel was everywhere parallel to the walls. At the junction, however, and particularly in the vicinity of the wall BC, the flow is not parallel to the wall but is curved. This curvature of flow will modify both assumptions. The water surface across the channel no longer will be level, and it is possible that the variation of pressure with depth

NOTE.—This paper by Edward H. Taylor, Jun. Am. Soc. C. E., was published in November, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1943, by Harold K. Palmer, M. Am. Soc. C. E.

[6] Senior Hydr. Engr., TVA, Hydraulic Laboratory, Norris, Tenn.

[6a] Received by the Secretary June 1, 1943.

may not be quite hydrostatic. Also, there may be some modification of pressure on the walls of the branch channel above section AC. From a further application of the momentum principle, the elevation of the water in the branch channel should be lower than that at the junction since there is a component of velocity normal to the main channel which is reduced to zero.

It would have been interesting if the author had developed his equations without the simplification introduced in Eq. 3 and had evaluated the term U from the experimental data. The results then might have been expressed in terms of a coefficient relating the experimental value of U to the calculated value.

Also it might have been enlightening if the pressures on the walls of the branch channel had been measured by means of piezometers. There seems to be no reason why the momentum principle should not give excellent results if it is applied correctly. Its proper application depends upon a complete knowledge of all the pressures having components in the direction of flow. The assumptions made in this case regarding pressure might well have been verified experimentally.

J. C. STEVENS,[7] M. AM. SOC. C. E.[7a]—It would be worth while for the reader of this excellent paper to look back 17 years and review the writer's short article[8] on similar flow characteristics at pipe intersections and particularly the discussions that followed, by Harold A. Thomas[9] and Julian Hinds,[10] Members, Am. Soc. C. E. In that article it was assumed that the pressure in each intersecting conduit would merge into a common value at the intersection. Momentum and energy relations were based on mean velocities. There followed from those assumptions a formula purporting to give the theoretical energy losses when n pipes of varying sizes join at a common junction at varying angles from which the water flowed away in a main.

Another formula gave expression to the theoretical losses resulting from n pipes of varying sizes discharging at right angles into a header of constant diameter. It made no difference in which direction the water was flowing—for example, the water flow could divide from the main to the several pipes and the same theoretical energy losses would result.

Those formulas will give approximate results. The errors are on the side of safety in that the losses indicated are on the excessive side. However, the formulas do illustrate the danger of simplifying assumptions if accuracy is important. Mr. Taylor was able to simplify his theoretical analysis by assuming that the depth in the branch remained constant to the wall of the main and that it was also equal to the depth in the main above the junction. Both of these assumptions are somewhat similar to the writer's assumption of uniform pressure at pipe intersections, and both are incorrect; yet approximate results may be obtained from the use of the author's formula and diagrams.

[7] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[7a] Received by the Secretary June 28, 1943.

[8] "Theoretical Energy Losses in Intersecting Pipes," by J. C. Stevens, *Engineering News-Record*, July 22, 1926, p. 140.

[9] *Ibid.*, p. 437.

[10] *Ibid.*, pp. 883–884.

The sources of error in these analyses are obvious. The water never has a uniform velocity distribution. After the junction a greatly distorted velocity distribution results. Both momentum and energy relationships based on a $\frac{Q}{A}$-mean are therefore considerably in error.

The force due to pressure on certain parts of the peripheries of the conduits at their junction, even in the case of uniform size conduits intersecting at right angles, cannot be expressed in known terms. In the paper cited,[8] this force was simply ignored as the easiest solution. Mr. Taylor assumes it to be equal and opposite to the pressure force in the branch, which, although preferable to ignoring it completely, still leaves much to be desired.

The Society's Committee on Hydraulic Research assigned the research project "Phenomenon of Intersecting Streams" to Professor M. P. O'Brien in 1936. Mr. Taylor's paper is the final paper on the open-channel phase of this subject. Research on intersecting pipes is being continued, although temporarily postponed in favor of more urgent war activities.

Flow in open-channel intersections was found to involve too many unknowns. Flow in intersecting pipes also contains too many complexities until more can be learned about how two streams mingle. As stated by the author, that phase is under study—first in its simplest form of mixing parallel streams. The next step will be to study the mixing of two nonparallel streams. Upon completion of these investigations, it is anticipated that studies on the flow characteristics of intersecting pipes can be resumed.

Almost any kind of research progresses slowly. The first steps open new vistas that must be explored before tangible results can be achieved on the original objective. Those engaged in teaching, of course, must make that their first duty, and research in such cases must be satisfied with what can be accomplished as extracurricular activities.

C. J. Posey,[11] Assoc. M. Am. Soc. C. E.[11a]—It should be emphasized that the data and conclusions of this paper are for flow deeper than the critical throughout—that is, with a ratio of velocity head to depth of less than one half in both branches of the stream. Although no statement to this effect appears in the paper, it is obviously true from the results, and is implicit in the author's statement that the elevations of the water surface in both channels could be varied by means of adjustable gates at the lower ends.

The data presented by the author justify the assumption customarily made in dealing with problems in combining or dividing flow at greater than critical depth—namely, that depths in the intersecting channels are substantially the same, with the division, in case of dividing flow, depending chiefly upon the backwater characteristics of the two branch channels. If greater refinement were needed, in some particular case, a model study patterned after the experiments reported in this paper should yield the desired information.

---

[11] Associate Prof., Hydraulics and Structural Eng., State Univ. of Iowa; and Research Engr., Iowa Inst. of Hydr. Research, Iowa City, Iowa.

[11a] Received by the Secretary July 16, 1943.

Hydraulic Translations p. 1-46 at end of vol.